El Control Parcial de sistemas caóticos

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Jornada Científica en Homenaje al

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Universidad Rey Juan Carlos
Móstoles, 12 de diciembre de 2019

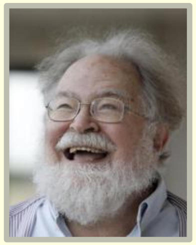
Control of chaos

Chaotic motion to regular motion

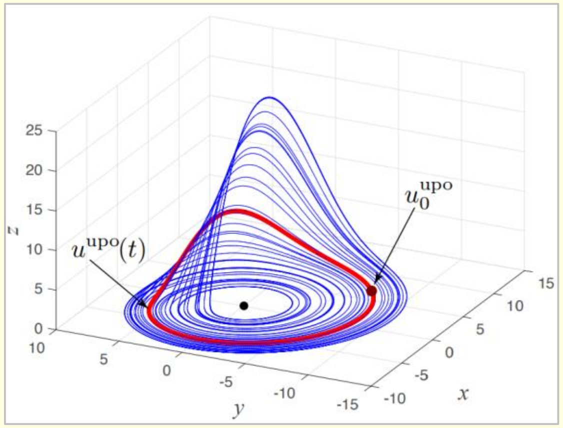
How?

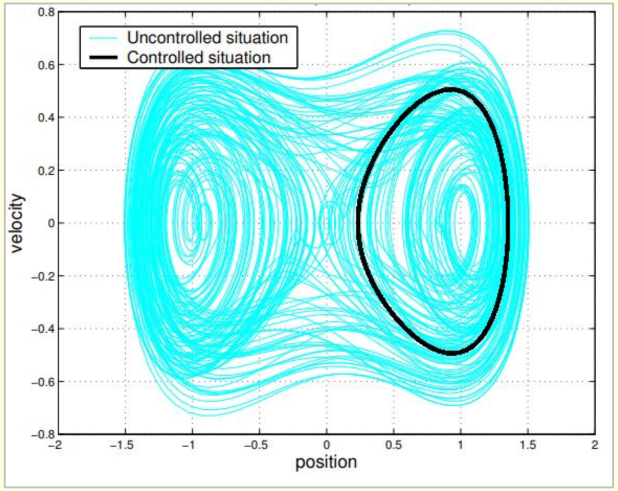
- *OGY method* (1990)



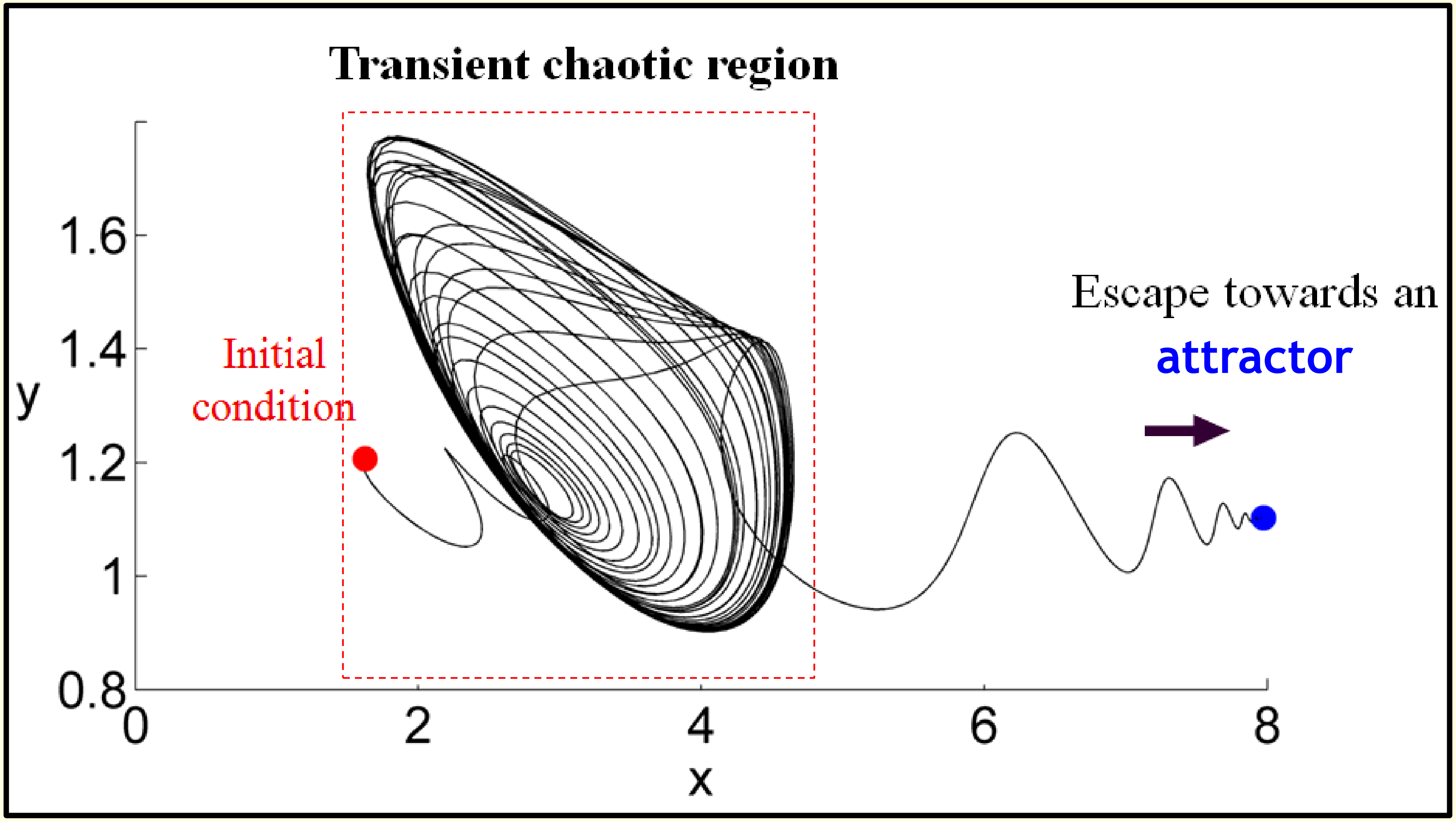


- *Delayed Feedback control* (1992)

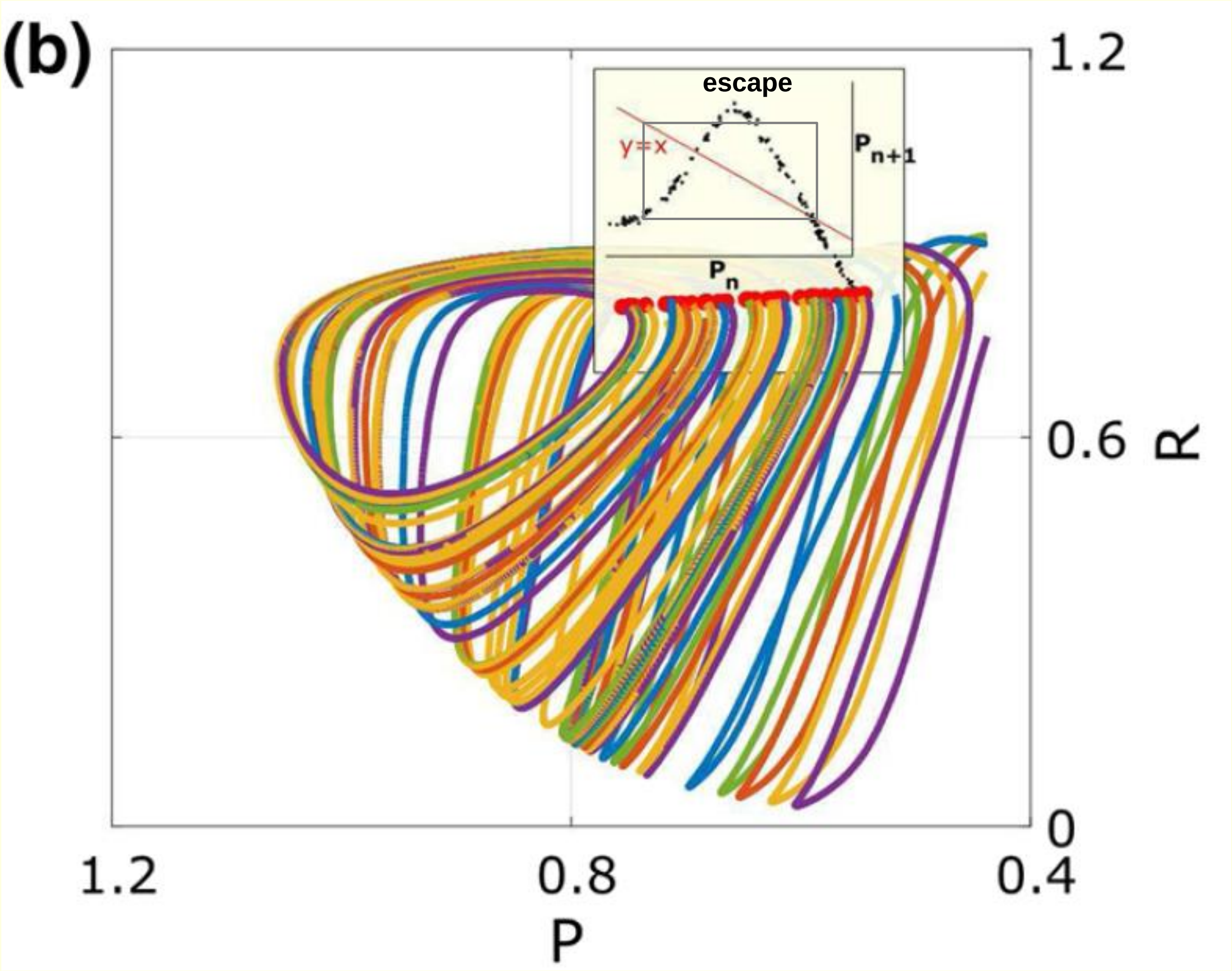




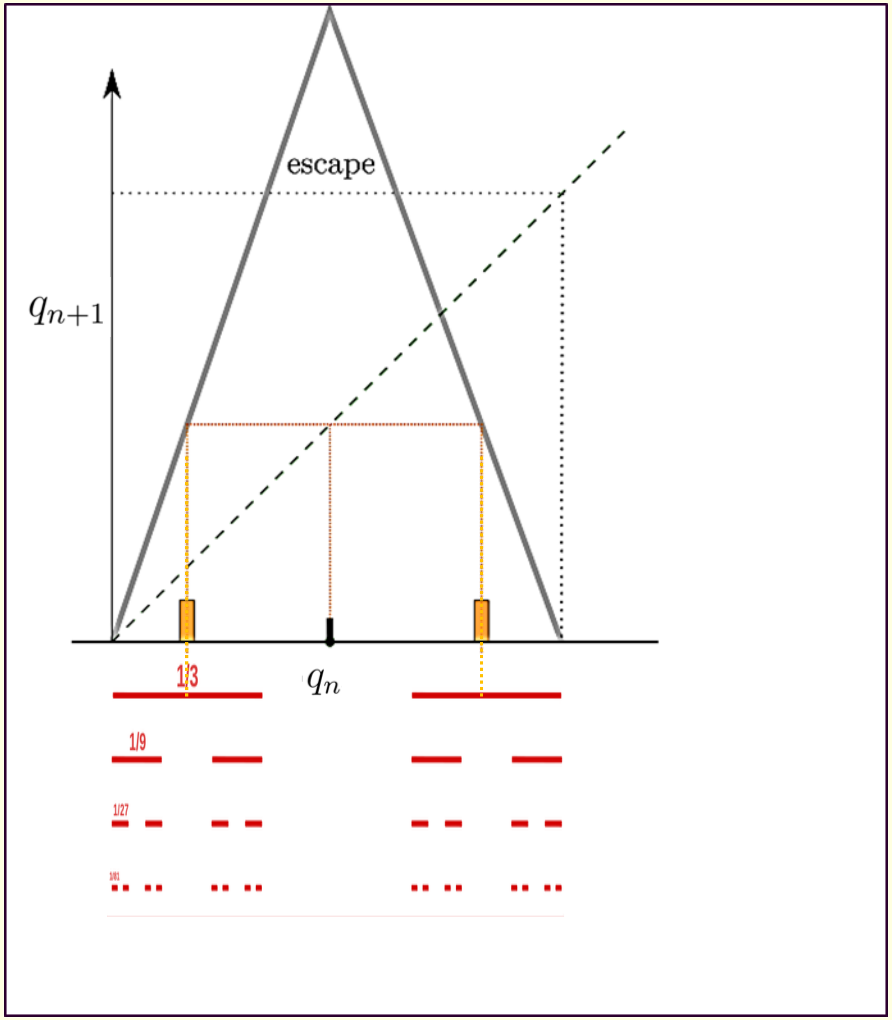
Transient chaos



Partial control fundaments

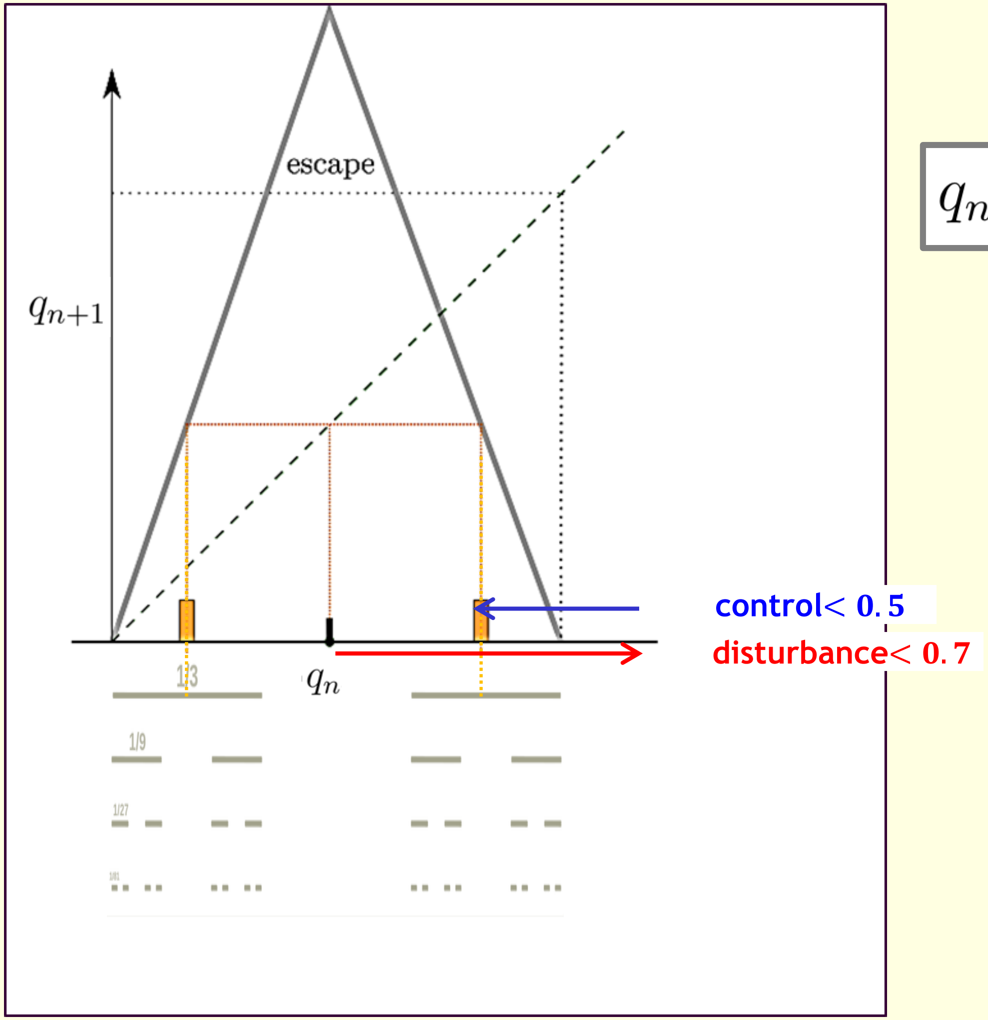


Partial control fundaments



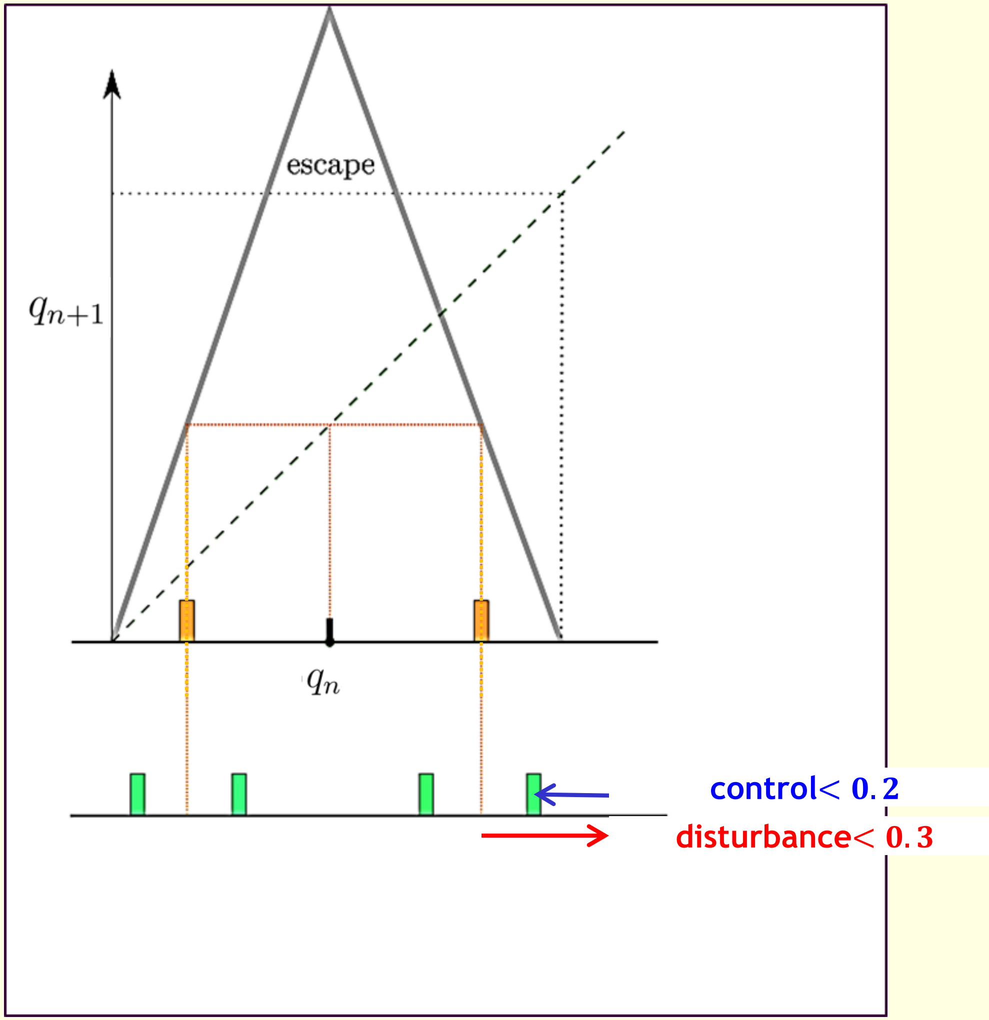
Cantor set of escapes

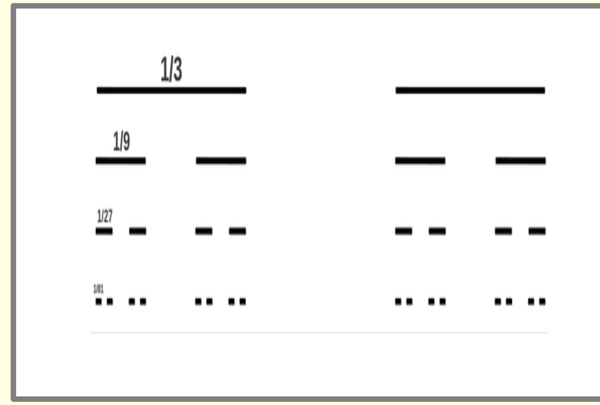
Partial control fundaments



$$\boxed{q_{n+1} = f(q_n) + \underbrace{\xi_n}_{\textit{disturbance}} + \underbrace{u_n}_{\textit{control}}}$$

Partial control fundaments





$$\boxed{u_0 < \xi_0}$$

Partial control algorithm

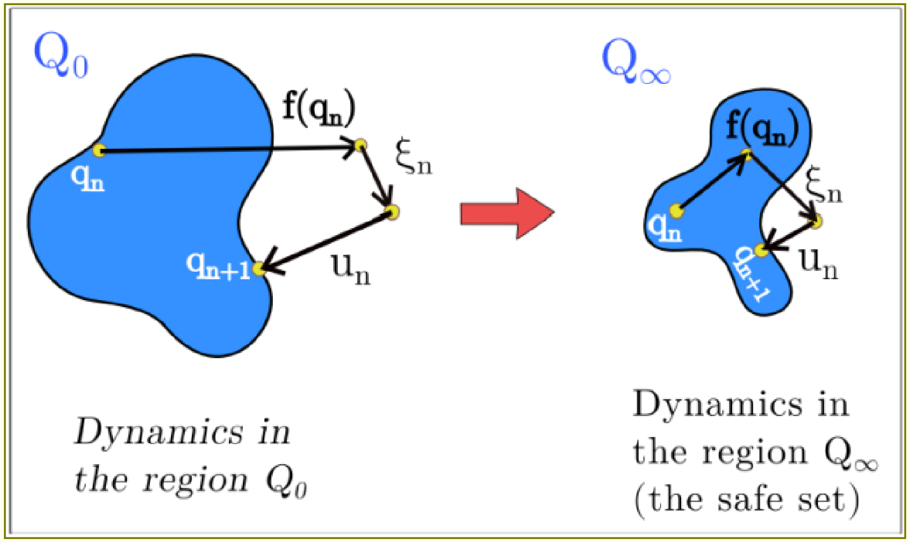
Dynamics in the map

Admissible trajectories

$$\boxed{q_{n+1} = f(q_n) + \underbrace{\xi_n}_{\text{disturbance}} + \underbrace{u_n}_{\text{control}}}$$

disturbance *control*

$$\boxed{|\xi_n| \leq \xi_0} \qquad \boxed{|u_n| \leq u_0}$$



Algorithm to find the safe set.

Controlled trajectories in the safe set

$$\boxed{u_0 < \xi_0}$$

Application to an ecological model



$$\frac{dR}{dt} = R\left(1 - \frac{R}{K}\right) - \frac{x_c y_c C R}{R + R_0}$$

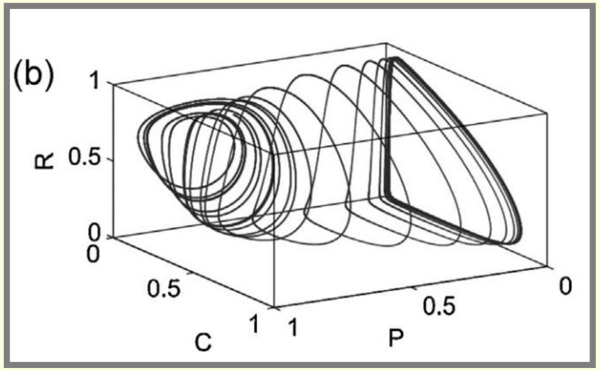
$$\frac{dC}{dt} = x_c C\left(\frac{y_c R}{R + R_0} - 1\right) - \psi(P)\frac{y_p C}{C + C_0}$$

$$\frac{dP}{dt} = \psi(P)\frac{y_p C}{C + C_0} - x_p P.$$

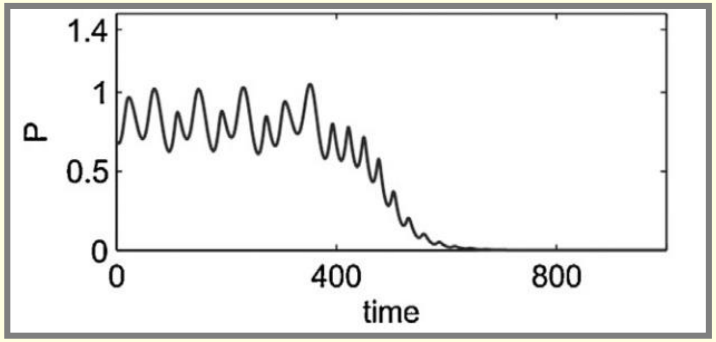
Duarte et al.

Chaos 58, 863-883, (2009)

Chaotic transient leads to the predators extinction



Phase space

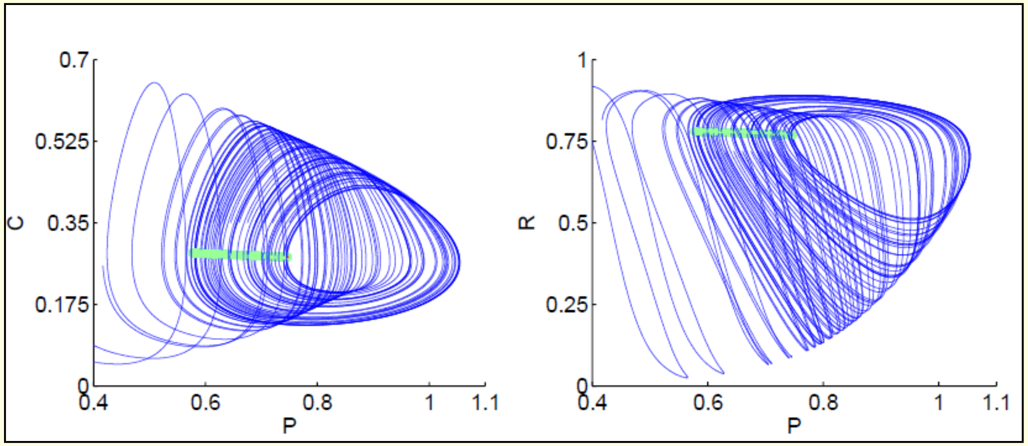


Time series of P

Application to an ecological model

Set of minima of P

$$\boxed{P_{n+1} = f(P_n)}$$

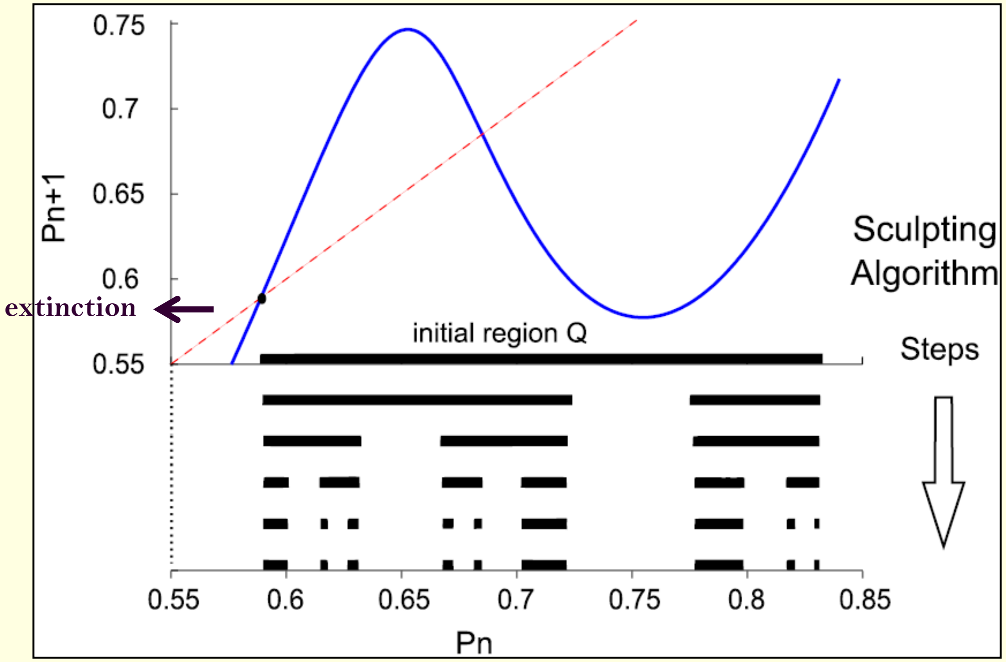


Return Map and the Safe Set

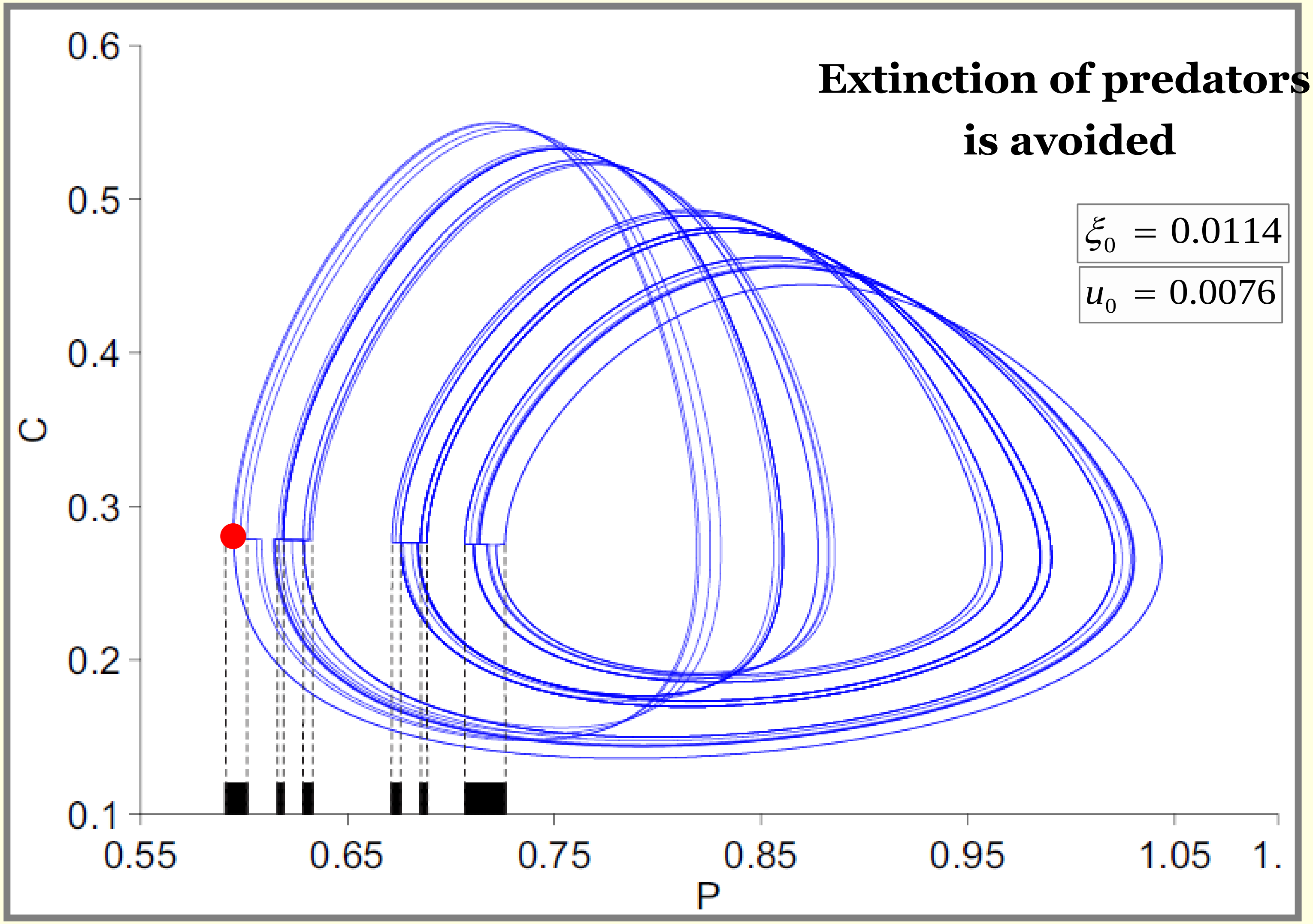
$$\boxed{P_{n+1} = f(P_n) + \xi_n + u_n}$$

$$\boxed{\xi_0 = 0.0114}$$

$$\boxed{u_0 = 0.0076}$$



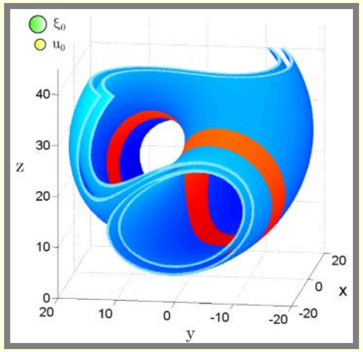
Application to an ecological model



Partial control development

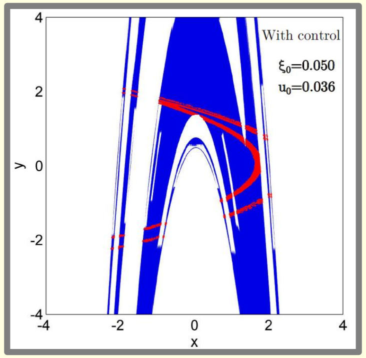
3D safe set

$$q_{n+1} = f(q_n) + \xi_n + u_n$$



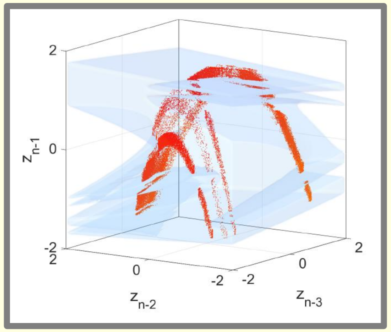
Parametric partial control

$$q_{n+1} = f(q_n, p + \xi_n + u_n)$$

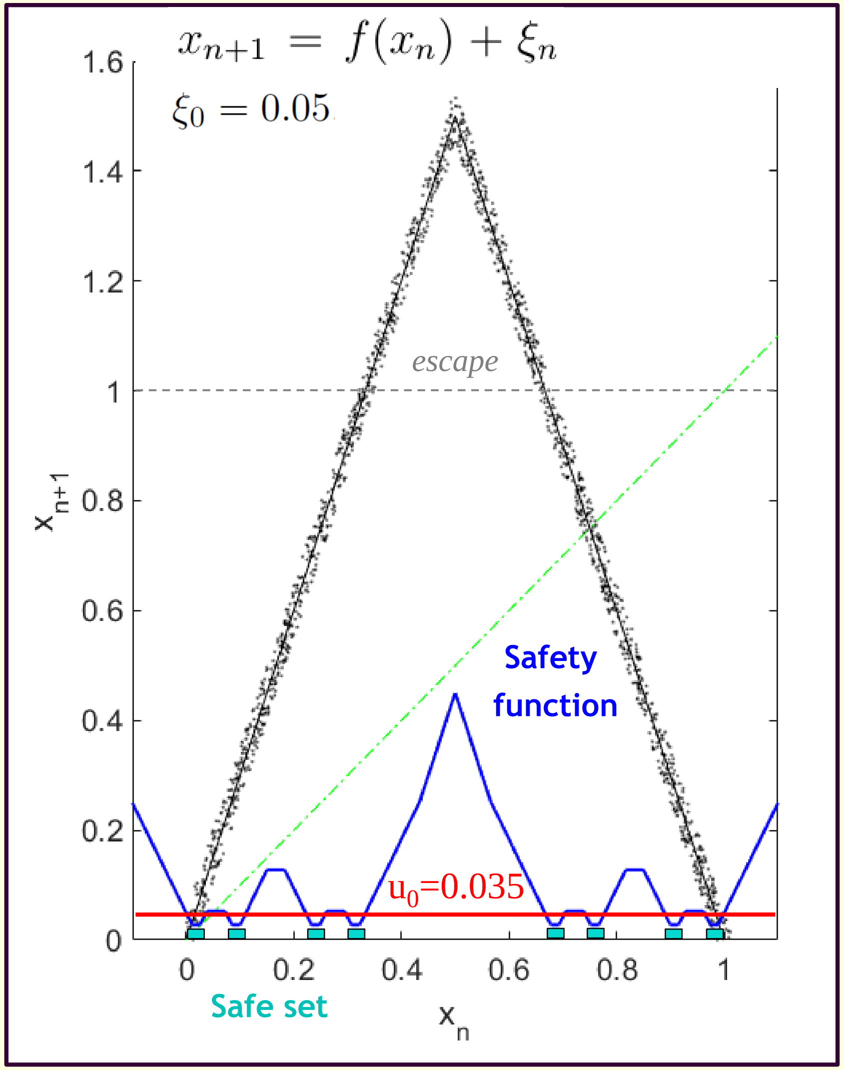


Time-delay partial control

$$x_n = f\,(x_{n-1}, x_{n-2}, \ldots) + \xi_n + u_n$$

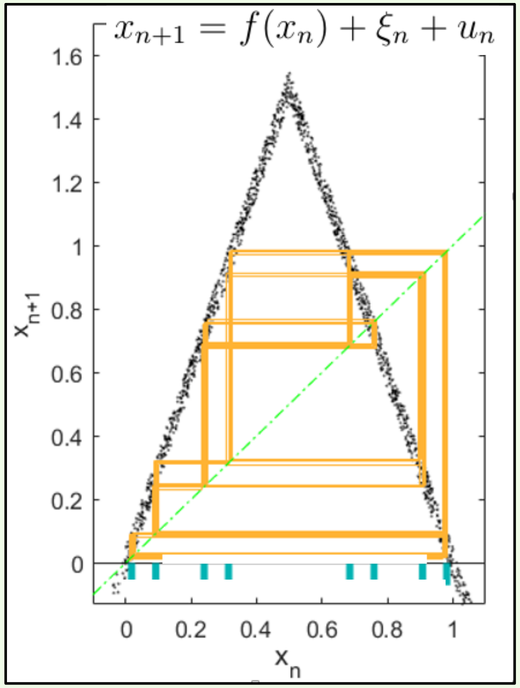


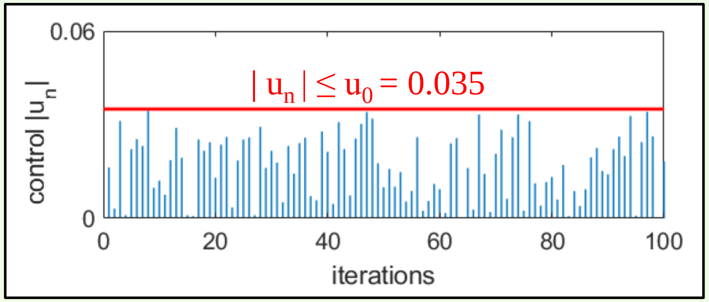
The safety function and the safe set



Q=[-0.1, 1.1]

Controlled trajectory





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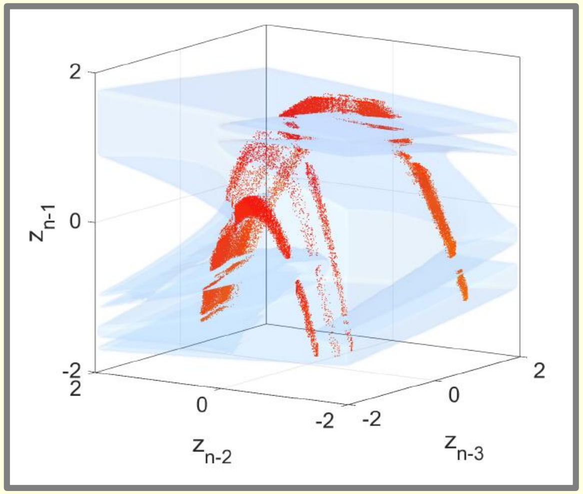
- **2008** S. Zambrano, M.A.F. Sanjuán and J.A. Yorke. *Partial control of chaotic systems.* Phys. Rev. E 77, 055201(R).
- **2009** S. Zambrano and M.A.F. Sanjuán. *Exploring partial control of chaotic systems.* Phys. Rev. E 79, 026217.
- **2010** J. Sabuco, S. Zambrano and M.A.F. Sanjuán. *Partial control of chaotic transients using escape times*. New J. Phys. 12, 113038.
- **2012** J. Sabuco, S. Zambrano, M.A.F. Sanjuán and J.A. Yorke. *Finding safety in partially controllable chaotic systems*. Commun. Nonlinear. Sci. Numer. Simul.17, 4274-4280, (2012).
- **2012** J. Sabuco, S. Zambrano, M.A.F. Sanjuán and J.A. Yorke. *Dynamics of partial control*. Chaos 22, 047507.
- **2013** M.Coccolo, J.M. Seoane, S. Zambrano and M.A.F. Sanjuán. *Partial control of escapes in chaotic scattering*. 23, 1350008.
- **2014** R. Capeáns, J. Sabuco and M.A.F. Sanjuán. *When less is more: Partial control to avoid extinction of predators in an ecological model*. Ecol. Complex. 19, 1-8.
- **2014** A.G. Lopéz, J. Sabuco, J.M. Seoane, J. Duarte, C. Januário and M.A.F. Sanjuán. *Avoiding healthy cells extinction in a cancer model*. J. Theor. Biol.349, 74-81.
- **2015** S. Das and J.A. Yorke. *Avoiding extremes using partial control*. J. Differ.Equations Appl.22, 217-234.
- **2016** R. Capeáns, J. Sabuco and M.A.F. Sanju´an. *Parametric partial control of chaotic systems*. Nonlinear Dyn. 2, 869-876.
- **2016** S. Naik and S.D. Ross. *Geometric aproaches in Phase Space Transport and Partial Control of Escaping Dynamics*. PhD Thesis. Virginia. Tech University.
- **2017** R. Capeáns, J. Sabuco, M.A.F. Sanjuán and J.A. Yorke. *Partially controlling transient chaos in the Lorenz equations*. Phil. Trans. R. Soc. A 375, 2088.
- **2017** A. Levi, J. Sabuco and M.A.F. Sanjuán. *When the firm prevents the crash: Avoiding market collapse with partial control*. Plos One . 12(8).
- **2017** V. Agarwal, J.Sabuco and B. Balachandran. *Safe regions with partial control of a chaotic system in the presence of white Gaussian noise*. IJBC 94, 3-1.
- **2018** R. Capeáns, J. Sabuco, and M.A. F. Sanjuán. *Partial control of delay-coordinate maps*. Nonlinear Dynamics 92, 1419-1429.
- **2019** R. Capeáns, J. Sabuco, and M.A. F. Sanjuán. *A new approach of the partial control method in chaotic systems*. Nonlinear Dynamics 98, 873-887.

$$x_n = f\,(x_{n-1}, x_{n-2}, \ldots) + \xi_n + u_n$$

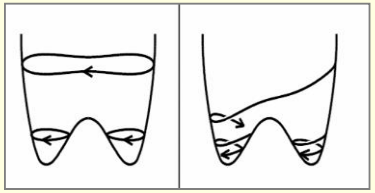


$$q[j] = f\big(q[i], \xi[s]\big) + u[i, s, j]$$

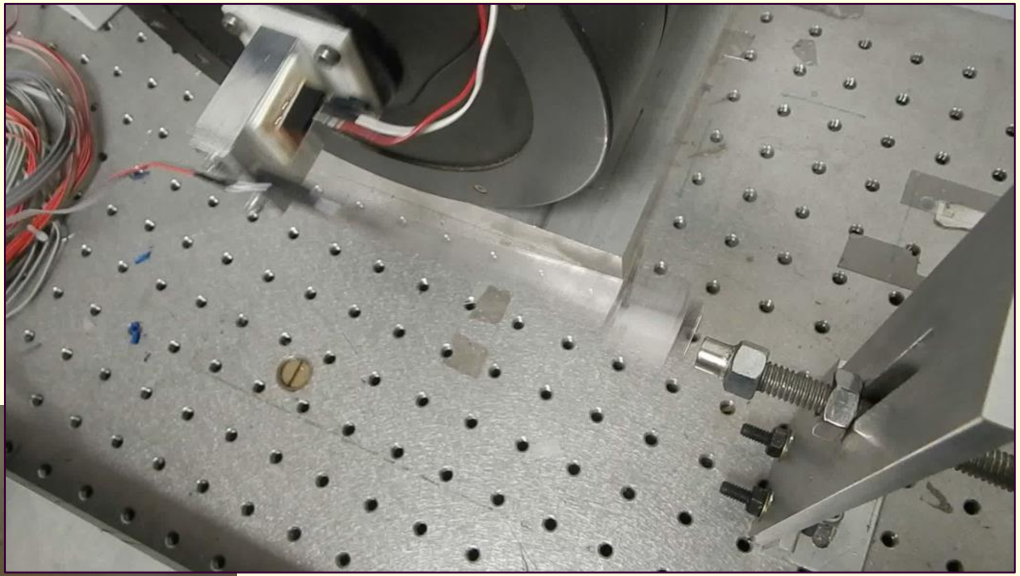
$$U_{k+1}[i] = \max_{1 \le s \le M_i} \Big(\min_{1 \le j \le N} \big(\max\big(u[i,s,j],\, U_k[j] \big) \big) \Big)$$

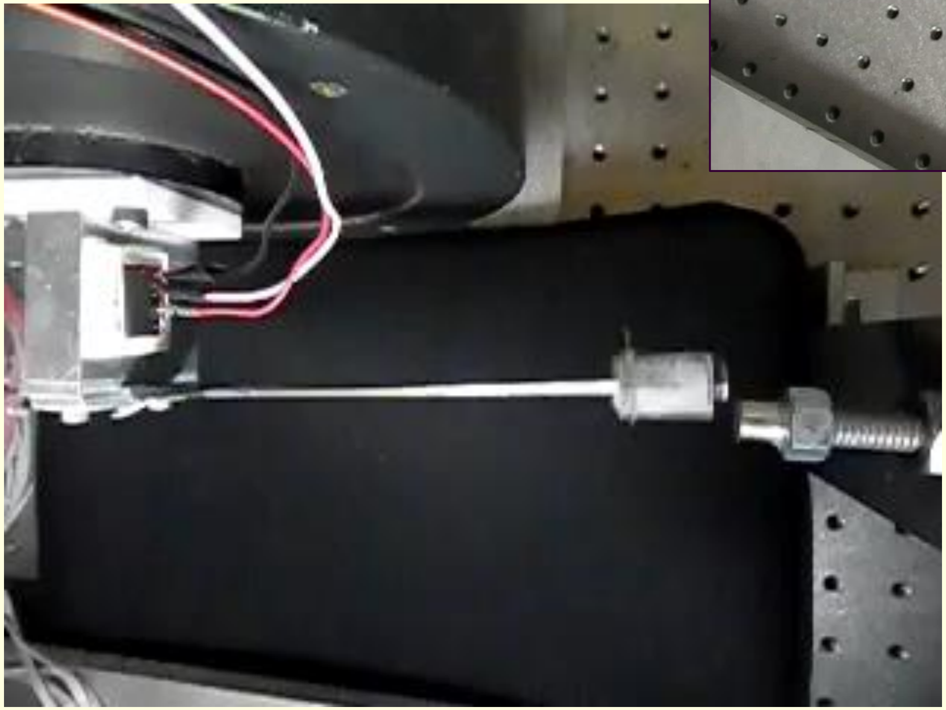
Application to the Duffing oscillator

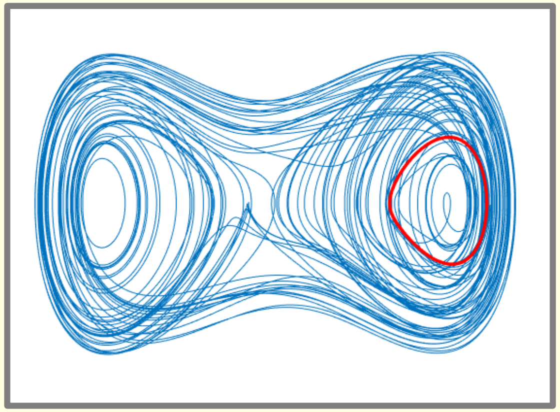
Duffing oscillator



$$\ddot{x} + \delta\dot{x} + \alpha x + \beta x^3 = F cos(wt)$$

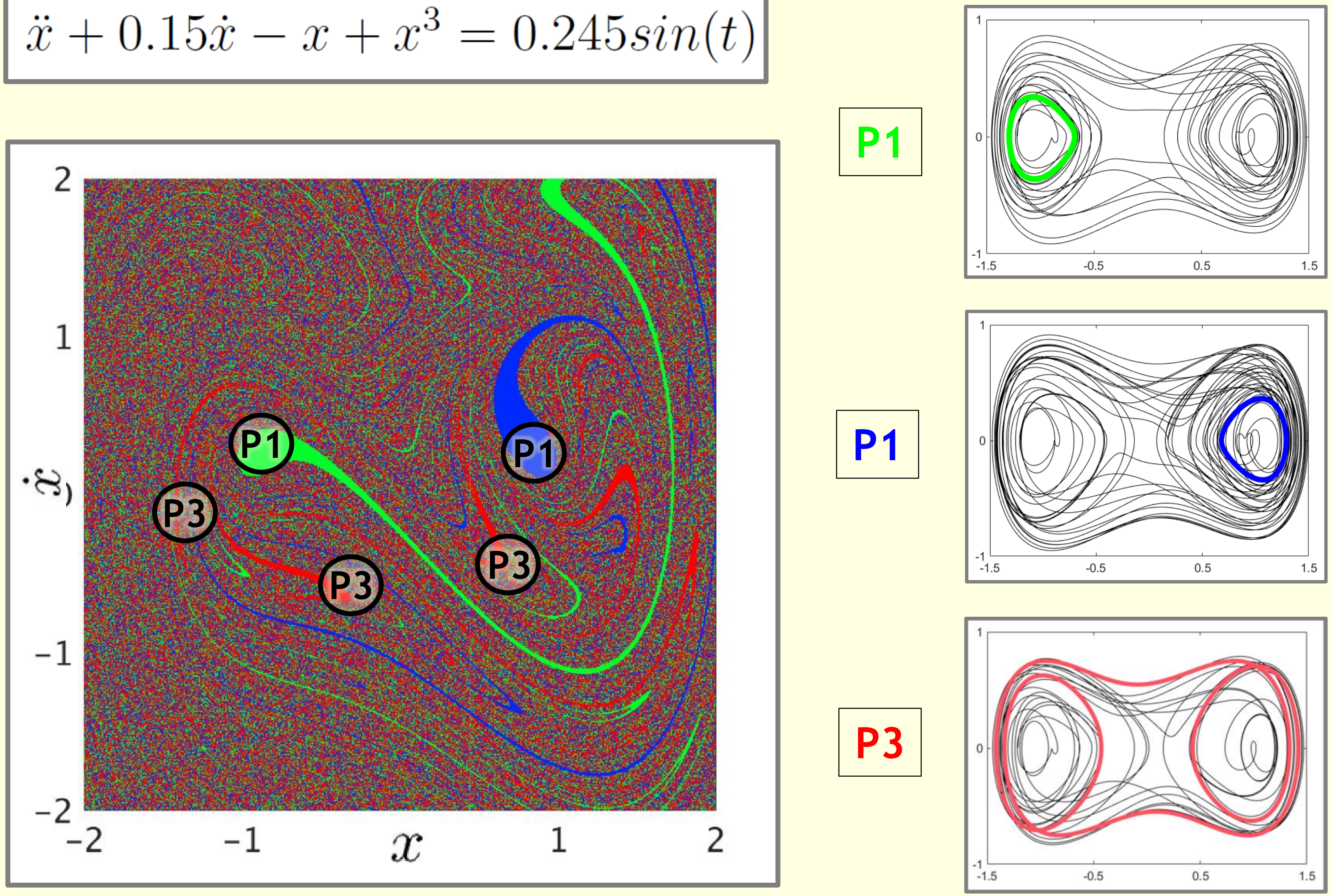






Application to the Duffing oscillator

$$\ddot{x} + 0.15\dot{x} - x + x^3 = 0.245 sin(t)$$



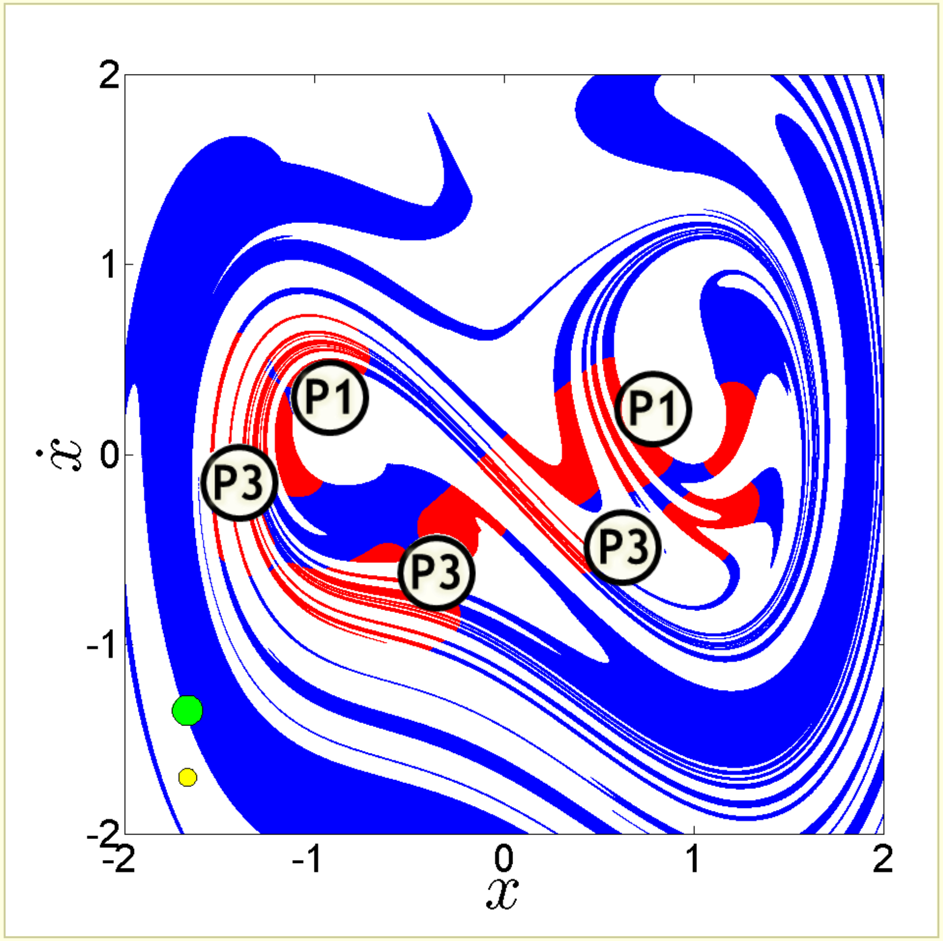
Application to the Duffing oscillator

$$\boxed{\ddot{x} + 0.15\dot{x} - x + x^3 = 0.245sin(t)}$$

Goal:

Avoid the periodic attractors

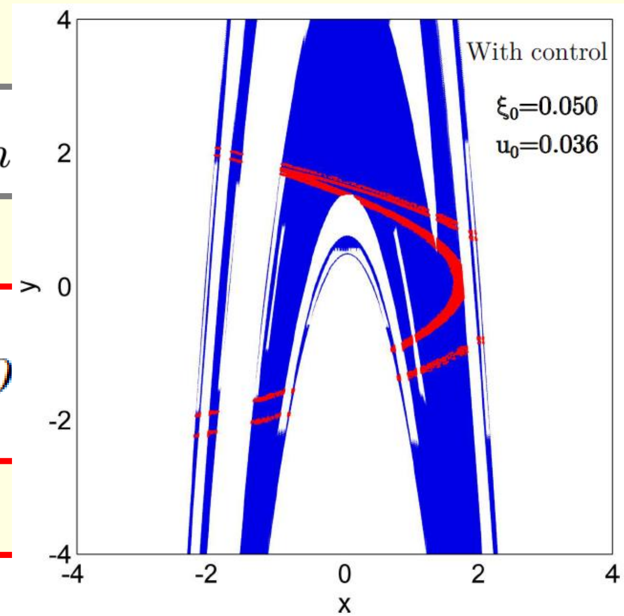
Safe set $\begin{cases} \xi_0 = 0.08 \\ u_0 = 0.0475 \end{cases}$

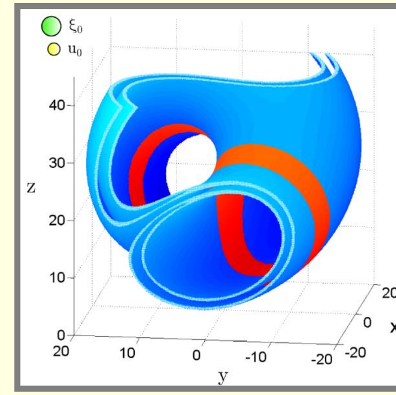


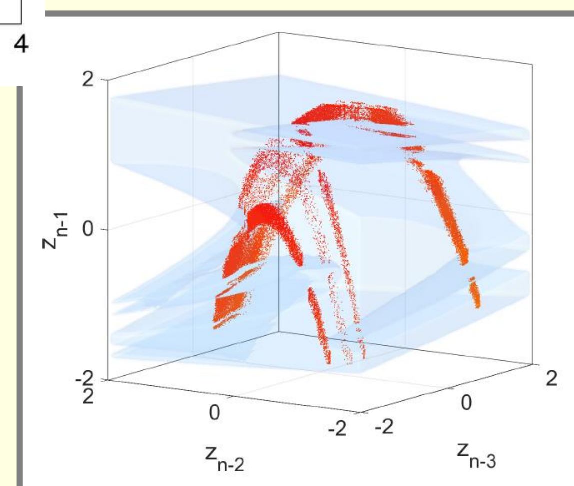
$$q_{n+1} = f(q_n) + \xi_n$$

$$q_{n+1} = f(q_n, p$$

$$x_n = f\,(x_{n-1}, x_{n-2}, \ldots, \; \xi_n \; u_n$$







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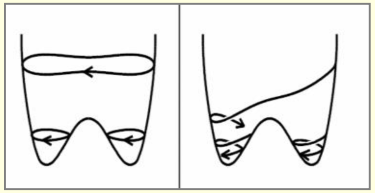
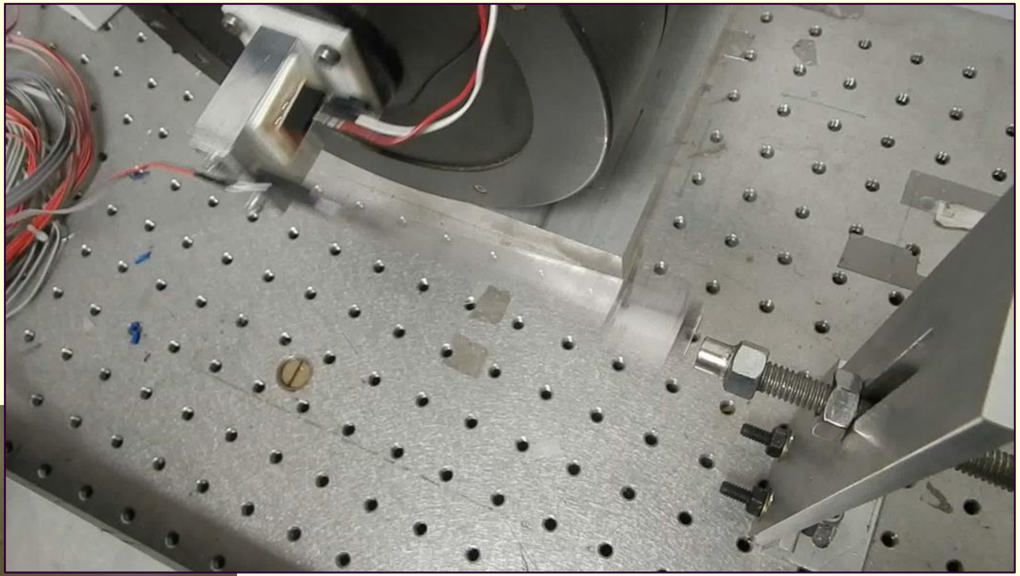
1- Introduction

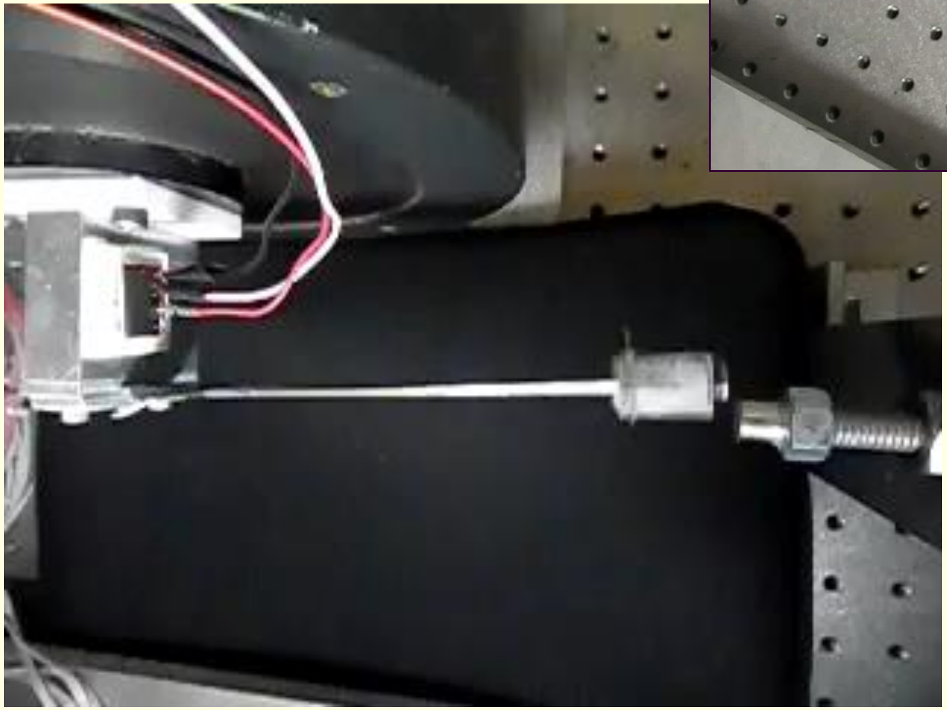
Experimental transient chaos

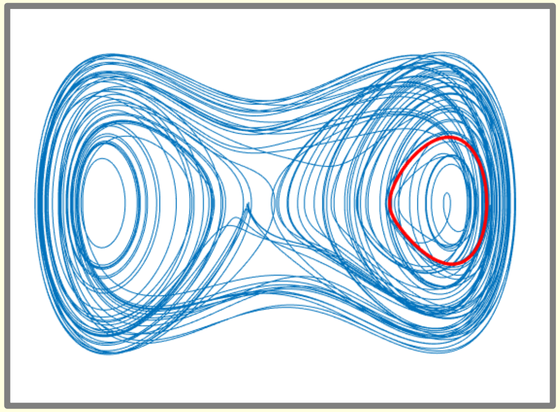
Duffing oscillator



$$\ddot{x} + \delta\dot{x} + \alpha x + \beta x^3 = F cos(wt)$$







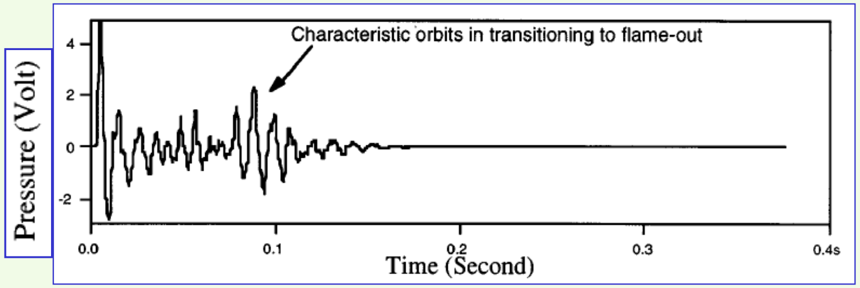
Transient chaos may involve an undesirable state

Visarath In, Mark L. Spano, Joseph D. Neff, William L. Ditto, C. Stuart Daw, K. Dean Edwards and Ke Nguyen. *Maintenance of chaos in a computational model of a thermal pulse combustor*. *Chaos* **7,** 605, 1997

$$\frac{d\tilde{P}}{dt} = \gamma\left\{\frac{1}{\tau_f} + \frac{1}{\tau_{\mathrm{HT}}} + \frac{1}{\tau_c}\right\} - \gamma\left\{\frac{Z_e}{\rho_0} + \frac{1}{\tau_{\mathrm{HT}}}\frac{T_0}{T_w}\right\}\tilde{T}.$$

$$\frac{d\tilde{T}}{dt} = \gamma\left\{\frac{1}{\tau_f} + \frac{1}{\tau_{\mathrm{HT}}} + \frac{1}{\tau_c}\right\}\frac{\tilde{T}}{\tilde{P}} - \left\{(\gamma-1)\frac{Z_e}{\rho_0} + \frac{1}{\tau_f} + \frac{\gamma T_0}{\tau_{\mathrm{HT}} T_w}\right\}\frac{\tilde{T}^2}{\tilde{P}}$$

Flame out in a thermal pulse combustor



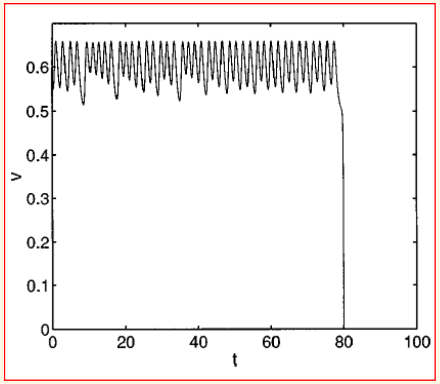
M. Dhamala and Y.C. Lai. *Controlling transient chaos in deterministic flows with applications to electrical power systems and ecology*. *Phys. Rev.* E **59**, 1646, 1999

$$\dot{\delta}_m = \omega,$$

$$M\dot{\omega} = -d_m\omega + P_m - E_m V Y_m \sin(\delta_m - \delta),$$

$$K_{qw}\dot{\delta} = -K_{qv2}V^2 - K_{qv}V + Q(\delta_m, \delta, V) - Q_0 - Q_1,$$

$$TK_{qw}K_{pv}\dot{V} = K_{pw}K_{qv2}V^2 + (K_{pw}K_{qv} - K_{qw}K_{pv})V + K_{qw}[P(\delta_m, \delta, V) - P_0 - P_1] - K_{pw}[Q(\delta_m, \delta, V) - Q_0 - Q_1].$$



Electrical power system collapse

2- Description of the partial control method

Partial control algorithm

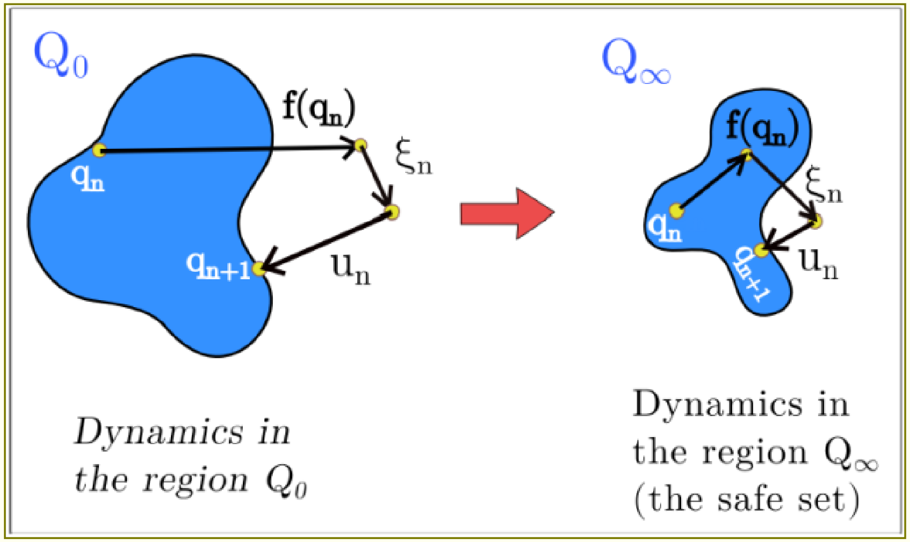
Dynamics in the map

Admissible trajectories

$$\boxed{q_{n+1} = f(q_n) + \xi_n + u_n}$$

disturbance *control*

$\boxed{|\xi_n| \leq \xi_0}$ $\boxed{|u_n| \leq u_0}$



Algorithm to find the safe set.

Controlled trajectories in the safe set

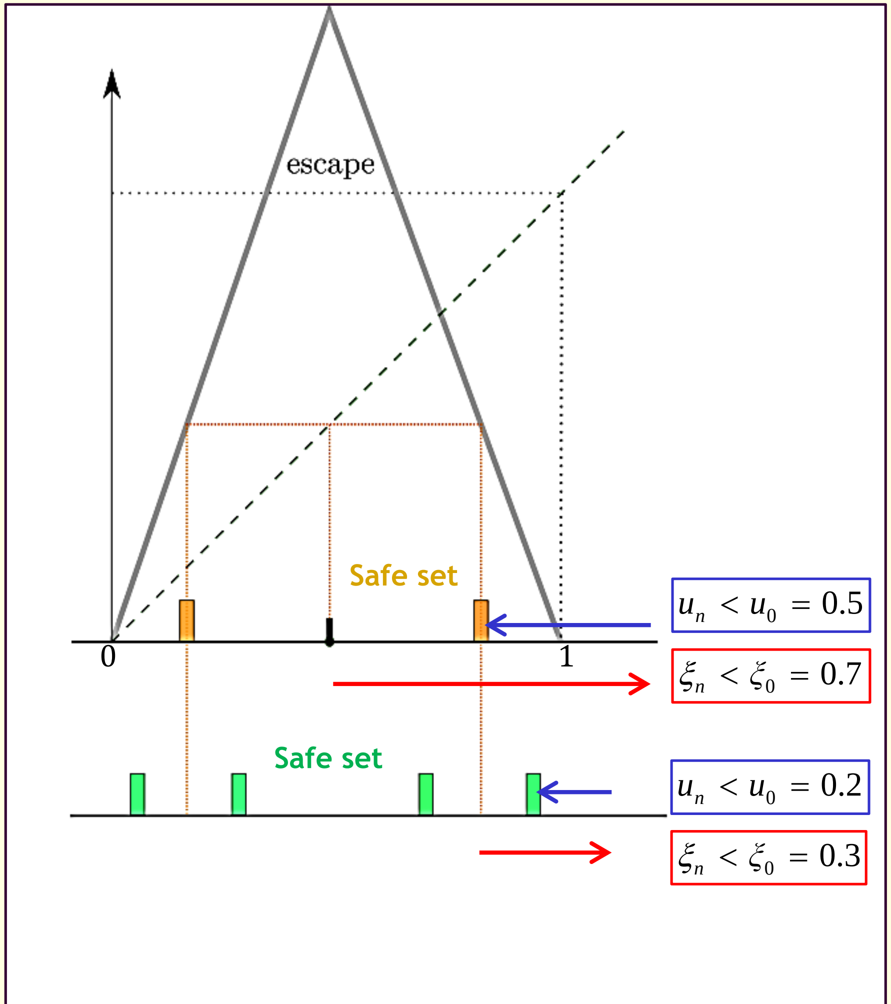
$$\boxed{u_0 < \xi_0}$$

Partial control: control < disturbance

$$x_{n+1} = f(x_n) + \xi_n + u_n$$

$Q_0=[0,1]$

Cantor-like set of the escape preimages



$$\boxed{u_0 < \xi_0}$$

Big disturbance case

Small disturbance case

3- Partial control to avoid a species extinction

R. Capeáns, J. Sabuco and M.A.F. Sanjuán. When less is more: Partial control to avoid extinction of predators in an ecological model. *Ecological Complexity* **19**, 1-8 (2014).

An ecological model

$$\frac{dR}{dt} = R\left(1 - \frac{R}{K}\right) - \frac{x_c y_c C R}{R + R_0}$$

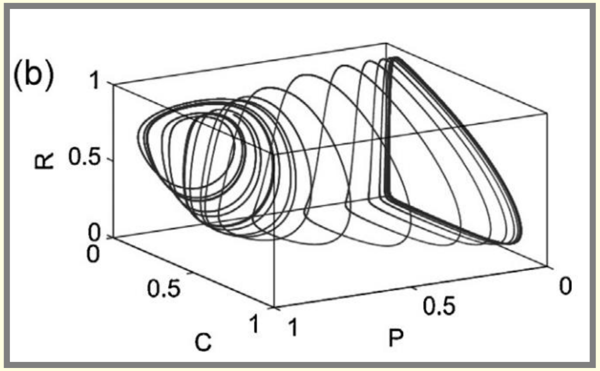
$$\frac{dC}{dt} = x_c C\left(\frac{y_c R}{R + R_0} - 1\right) - \psi(P)\frac{y_p C}{C + C_0}$$

$$\frac{dP}{dt} = \psi(P)\frac{y_p C}{C + C_0} - x_p P.$$

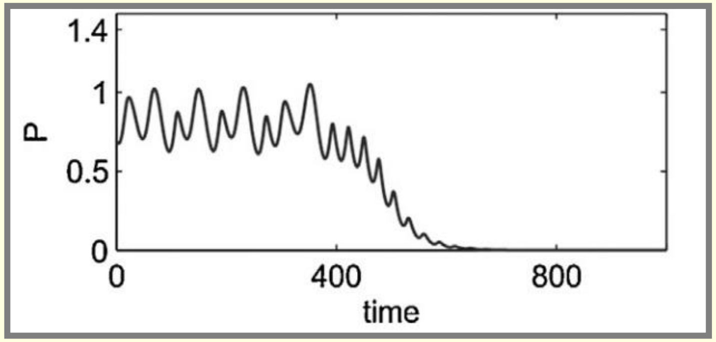
Duarte et al.

Chaos 58, 863-883, (2009)

Chaotic transient leads to the predators extinction



Phase space

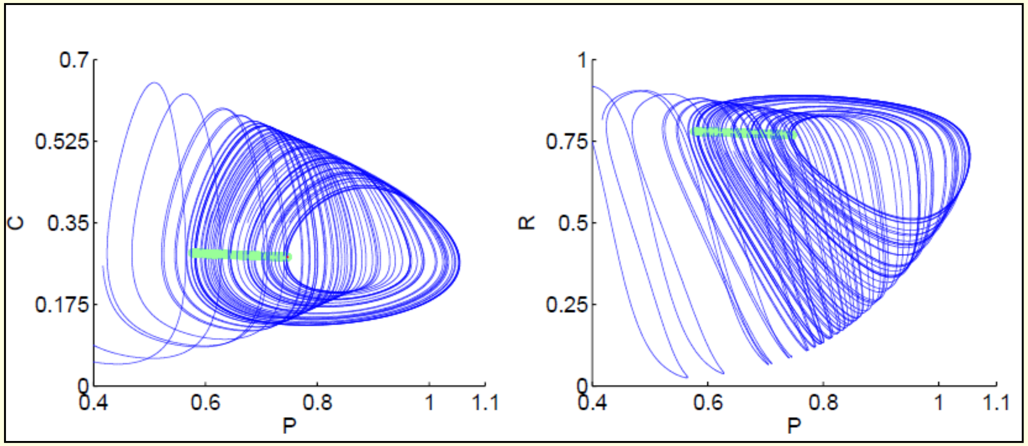


Time series of P

To build a map and implement the partial control

Set of minima of P

$$P_{n+1} = f(P_n)$$

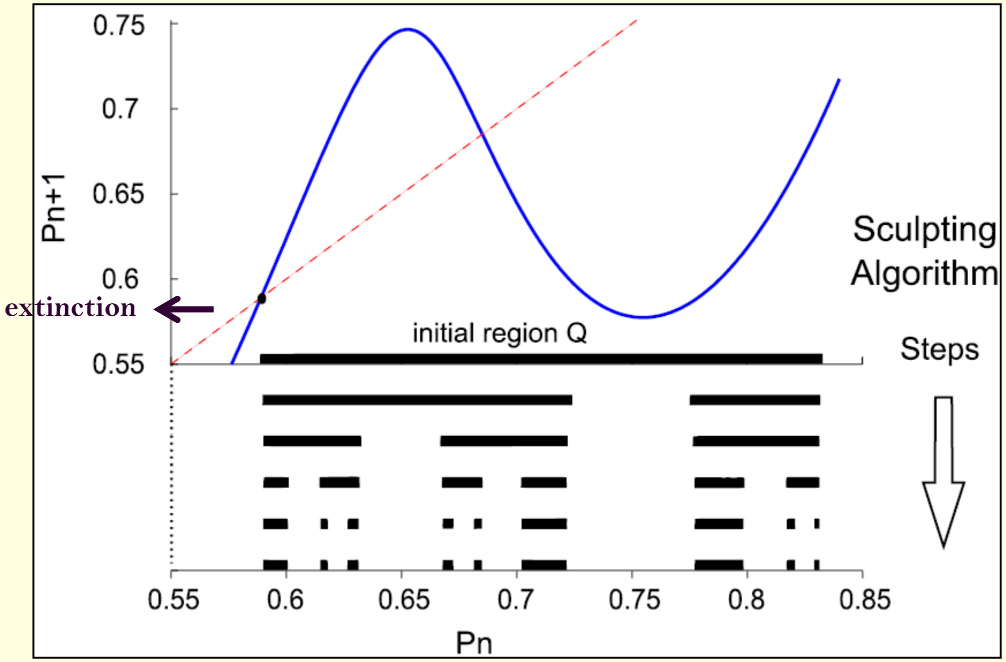


Return Map and the Safe Set

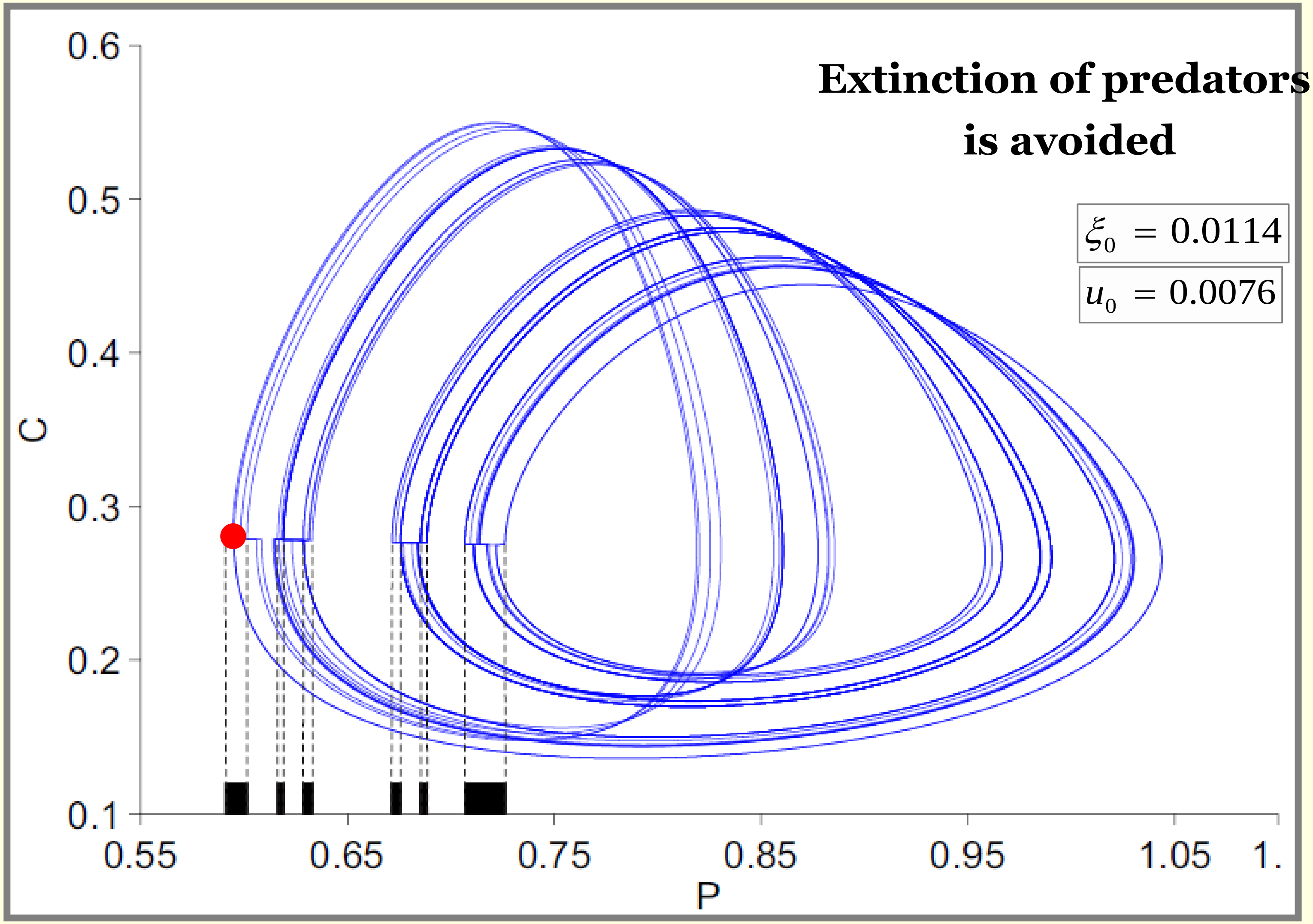
$$P_{n+1} = f(P_n) + \xi_n + u_n$$

$$\xi_0 = 0.0114$$

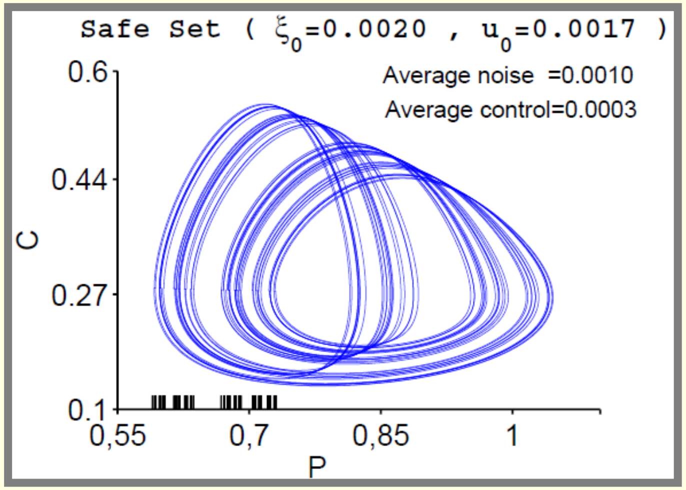
$$u_0 = 0.0076$$

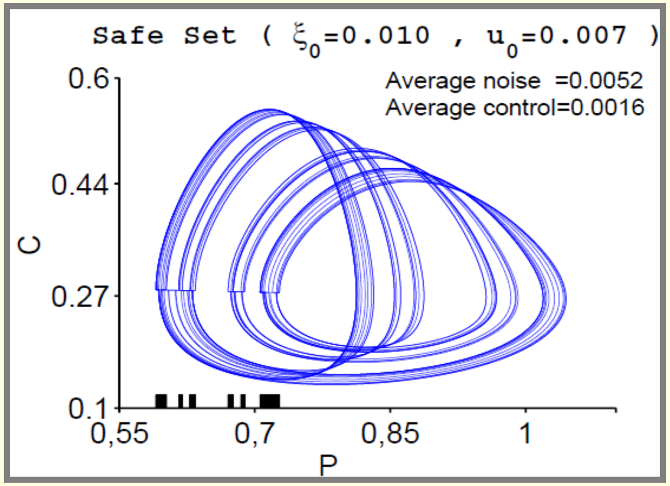


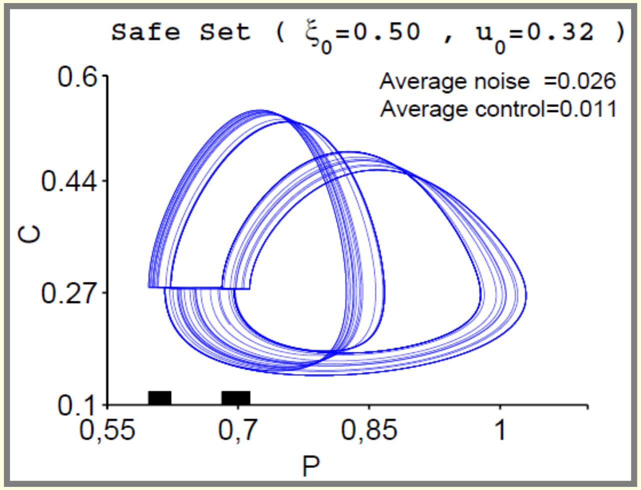
Partial control in the phase space



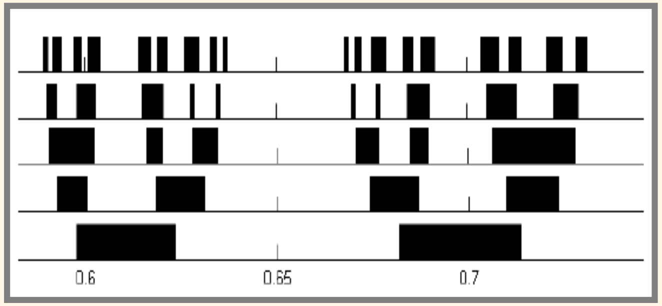
Different disturbance and control







Safe sets pattern



ξ_0

Increase

4- Controlling chaos in the Lorenz system

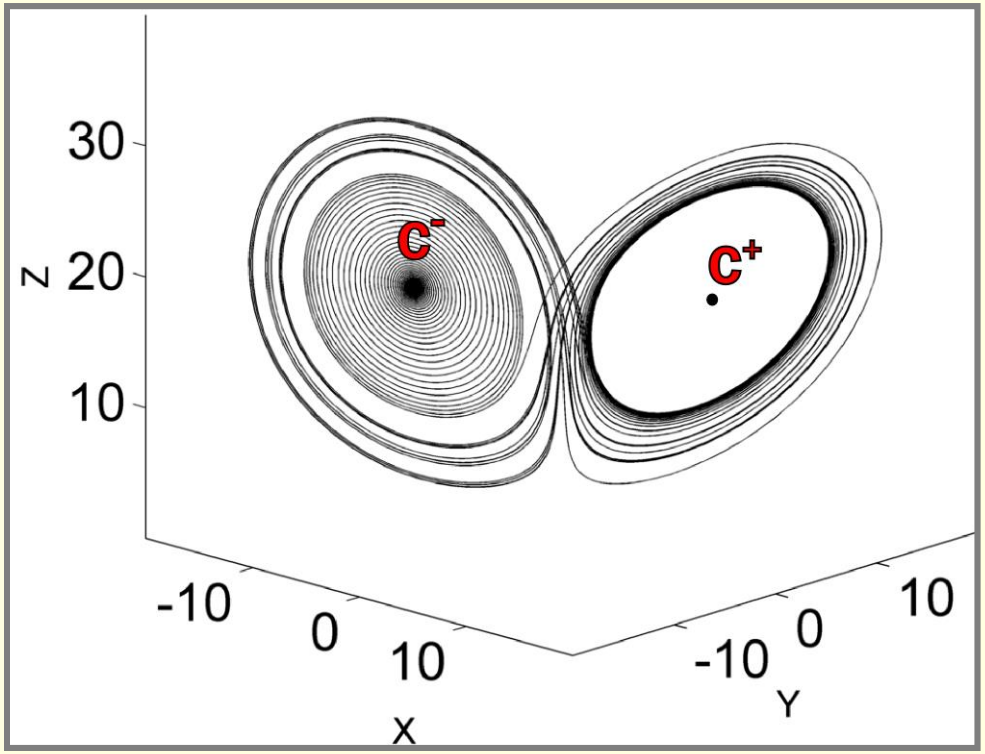
R. Capeáns, J. Sabuco, M. A. F. Sanjuán and J. A.Yorke. Partially controlling transient chaos in the Lorenz equations. *Philosophical Transactions of the Royal Society A* **375**, 2088 (2017).

Transient chaos in the Lorenz system

$$\dot{x} = -\sigma x + \sigma y$$
$$\dot{y} = -xz + rx - y$$
$$\dot{z} = xy - bz.$$

σ=10 *b=8/3* *r=20*

Transient chaos



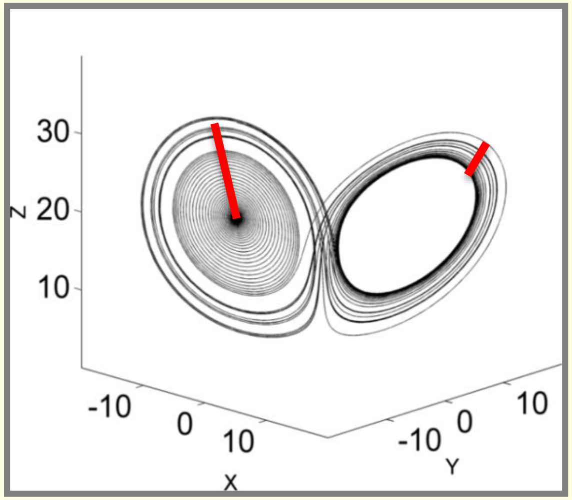
Goal: avoid the attractors:

$$C^- = (-7.12, -7.12, 19)$$
$$C^+ = (7.12, 7.12, 19)$$

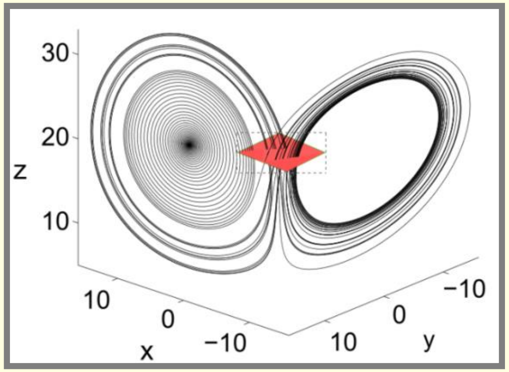
To build a map in the Lorenz system

Different ways to build a map : $\boxed{q_{n+1} = f(q_n)}$

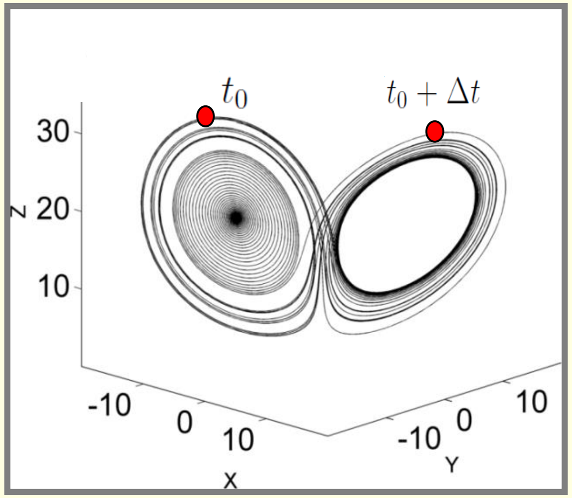
1D map with maxima of z



2D map with Poincaré section

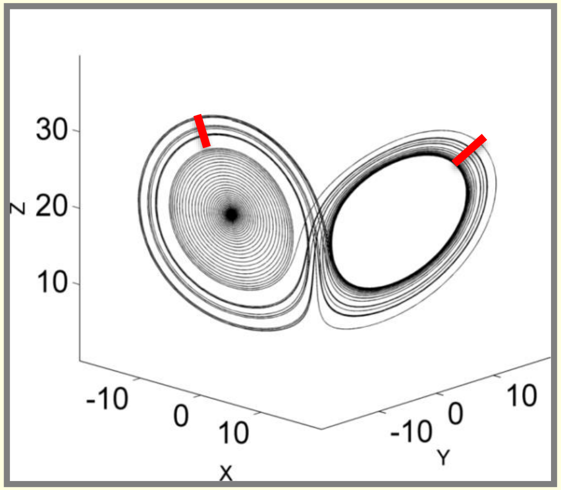


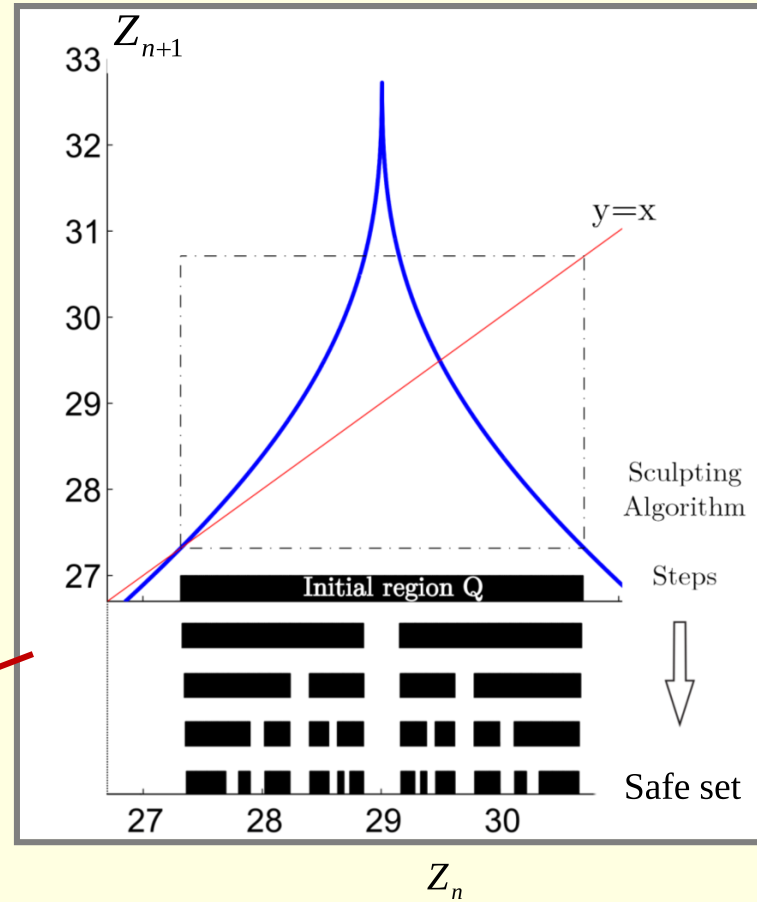
3D map with $\mathbf{x}(t_0) \rightarrow \mathbf{x}(t_0 + \Delta t)$

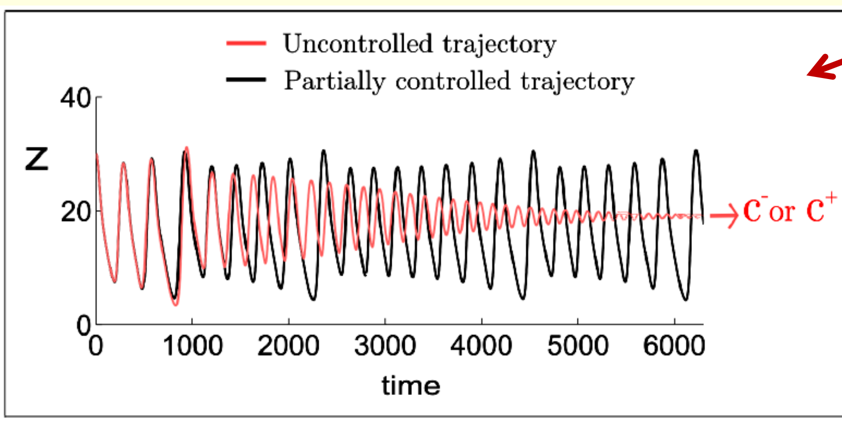


1D safe set

1D map with the maxima of z







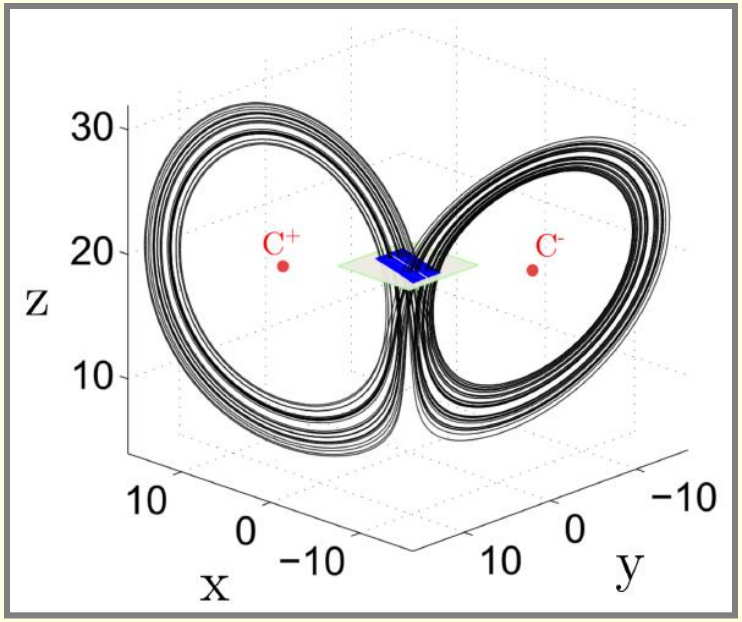
Disturbance $\xi_0 = 0.09$

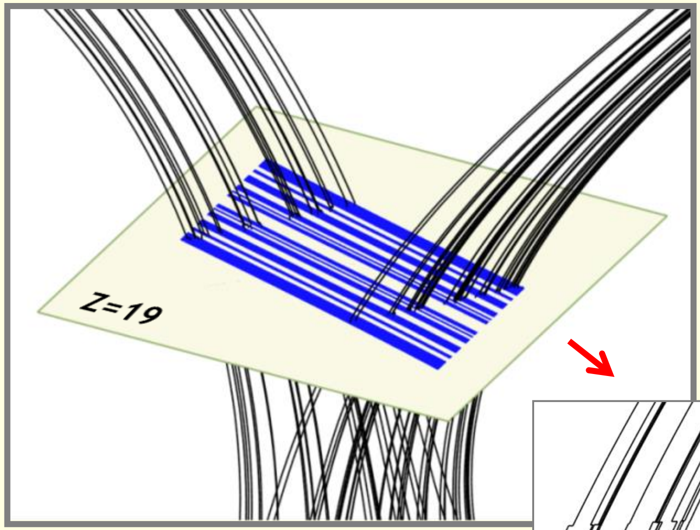
Control $u_0 = 0.06$

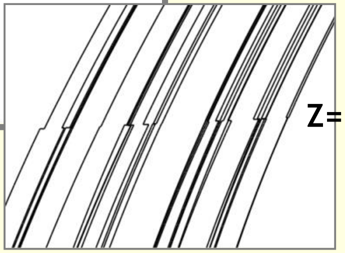
2D safe set

Discretization with a Poincaré section

Q_0 $\boxed{x \in [-3,3]}$ $\boxed{y \in [-3,3]}$ $\boxed{z = 19}$







$\boxed{\xi_0 = 0.09}$ $\boxed{u_0 = 0.06}$

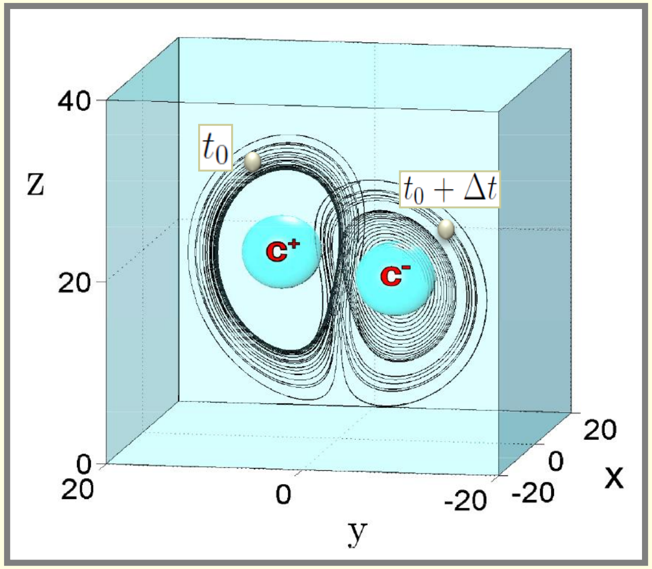
3D Map

$$X(t), Y(t), Z(t) \to X(t+\Delta t), Y(t+\Delta t), Z(t+\Delta t)$$

- **Initial region Q is the cube**

$x \in [-20, 20]$ $y \in [-20, 20]$ $z \in [0, 40]$

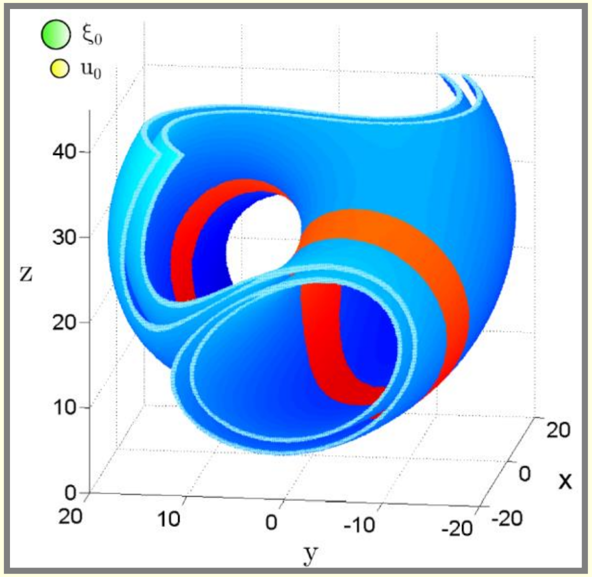
- **Balls centered in C+ and C- (removed)**



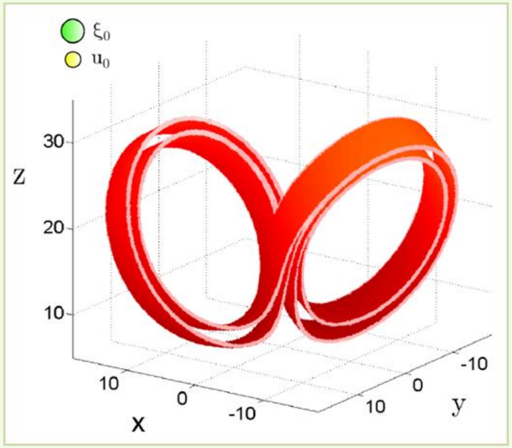
Safe set

Asymptotic safe set

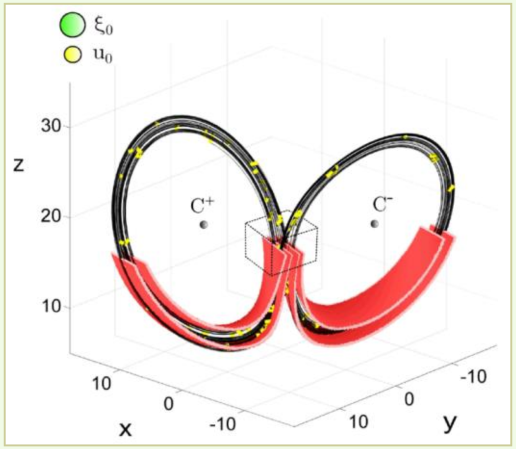
$\xi_0 = 1.5$ $u_0 = 1.0$ $\Delta t = 1.2$

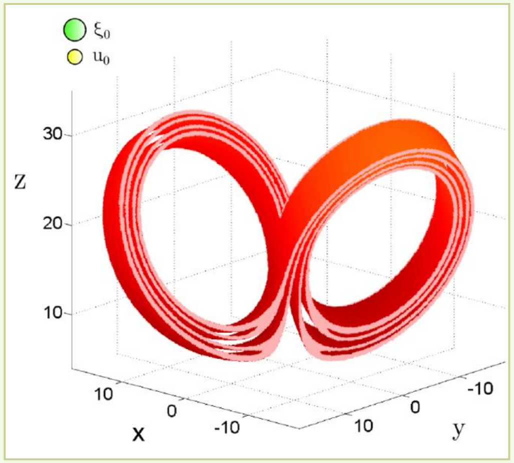


3D asymptotic safe set $u_0 = 1 \quad \xi_0 = 1.5$

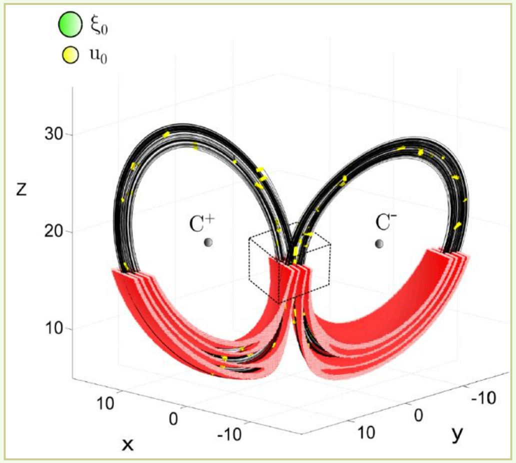


$\Delta t = 1.2$





$\Delta t = 1.8$



5- A different application of partial control

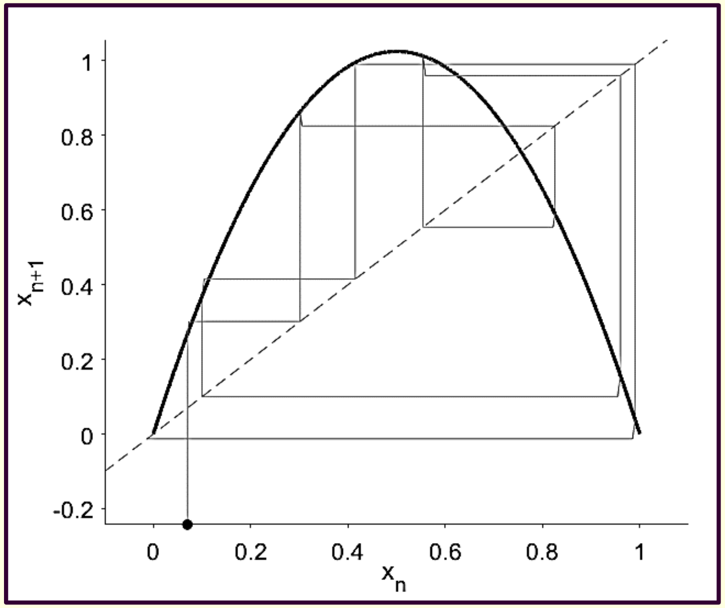
R. Capeáns, J. Sabuco and M. A. F. Sanjuán. Escaping from a chaotic saddle in the presence of noise. *International Journal of Nonlinear Dynamics and Control* **1**, 1-8, (2017).

Escape or not: the logistic map

$$x_{n+1} = 4.1x_n(1 - x_n) + \xi_n$$

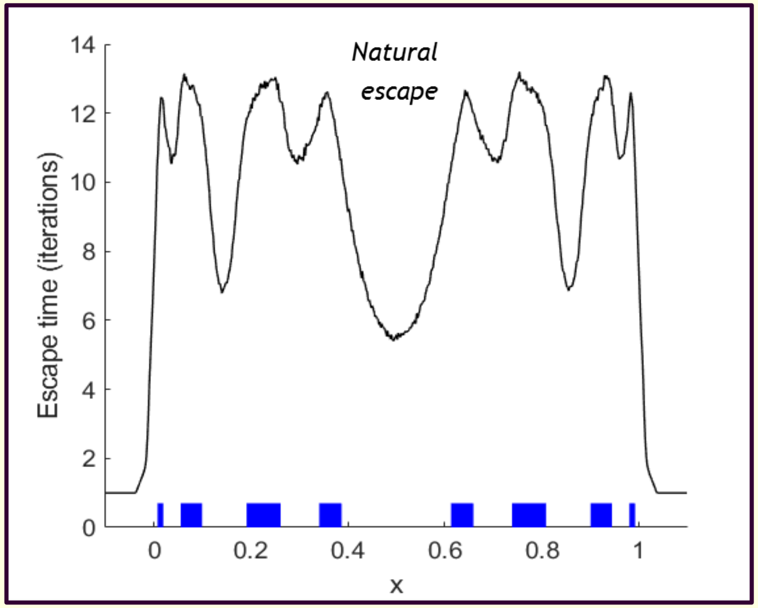
Escape from the safe set to reduce the escape time?

Logistic map without control



$$|\xi_n| \leq \xi_0 = 0.03$$

Correlation: escape time and safe set

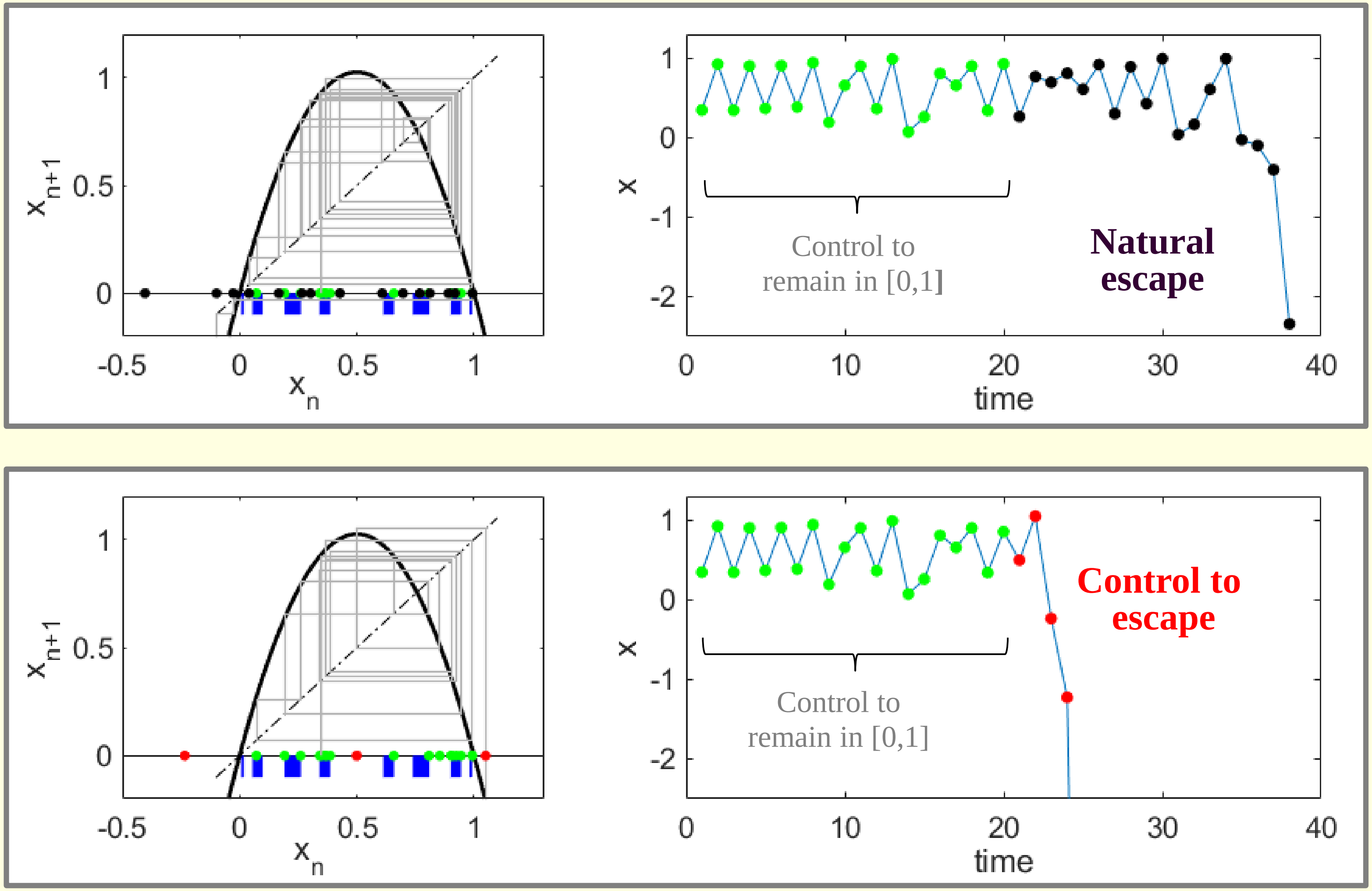


Q_0=[0,1]

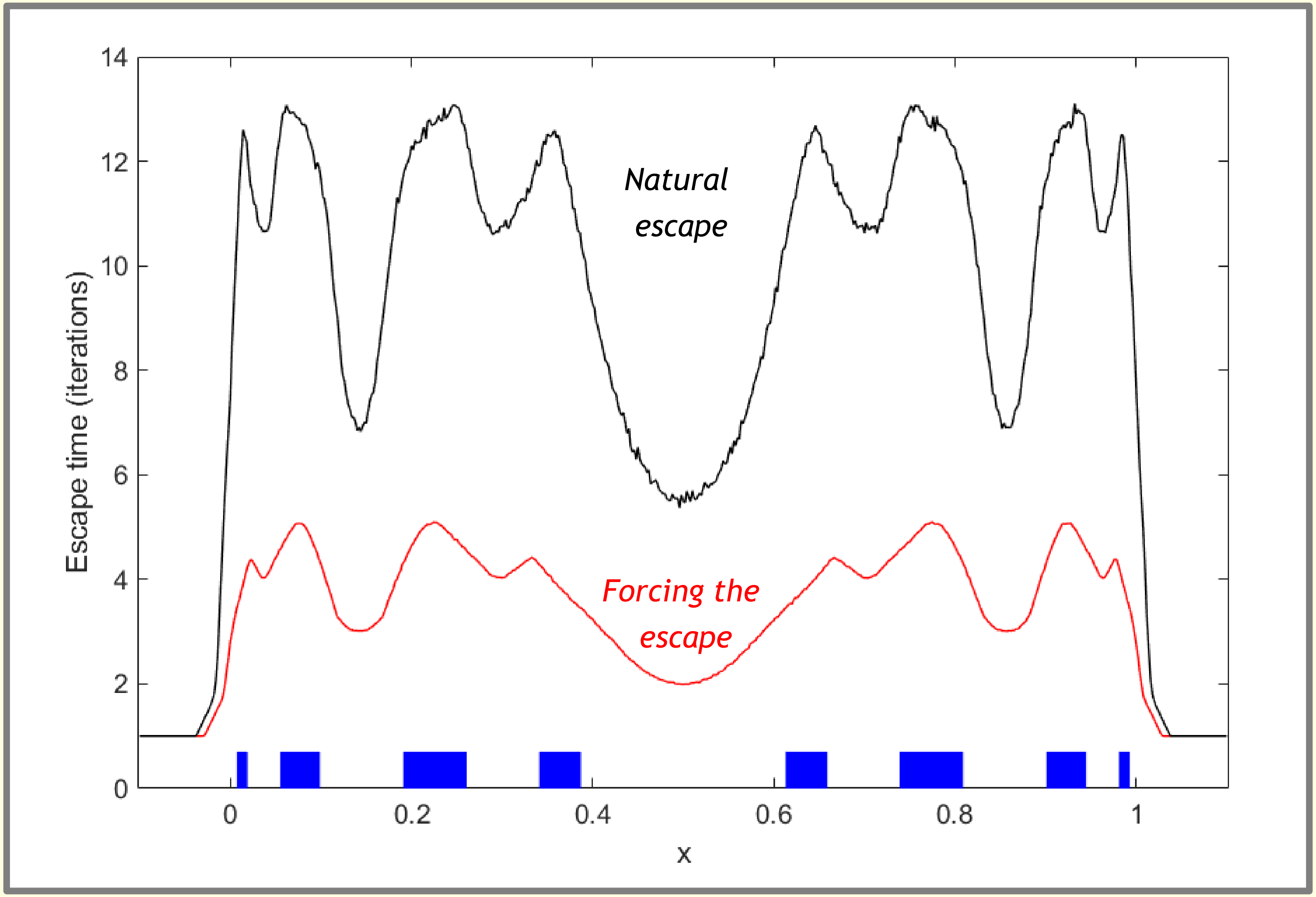
Safe set: $\begin{cases} \xi_0 = 0.03 \\ u_0 = 0.02 \end{cases}$

Escape or not: the logistic map (comparison)

$\boxed{\xi_0 = 0.03}$ $\boxed{u_0 = 0.02}$



Escape or not: the logistic map (comparison)



6- When the disturbance affects a parameter

R. Capeáns, J. Sabuco and M. A. F. Sanjuán. Parametric partial control of chaotic systems. *Nonlinear Dynamics* **2**, 869-876, (2016).

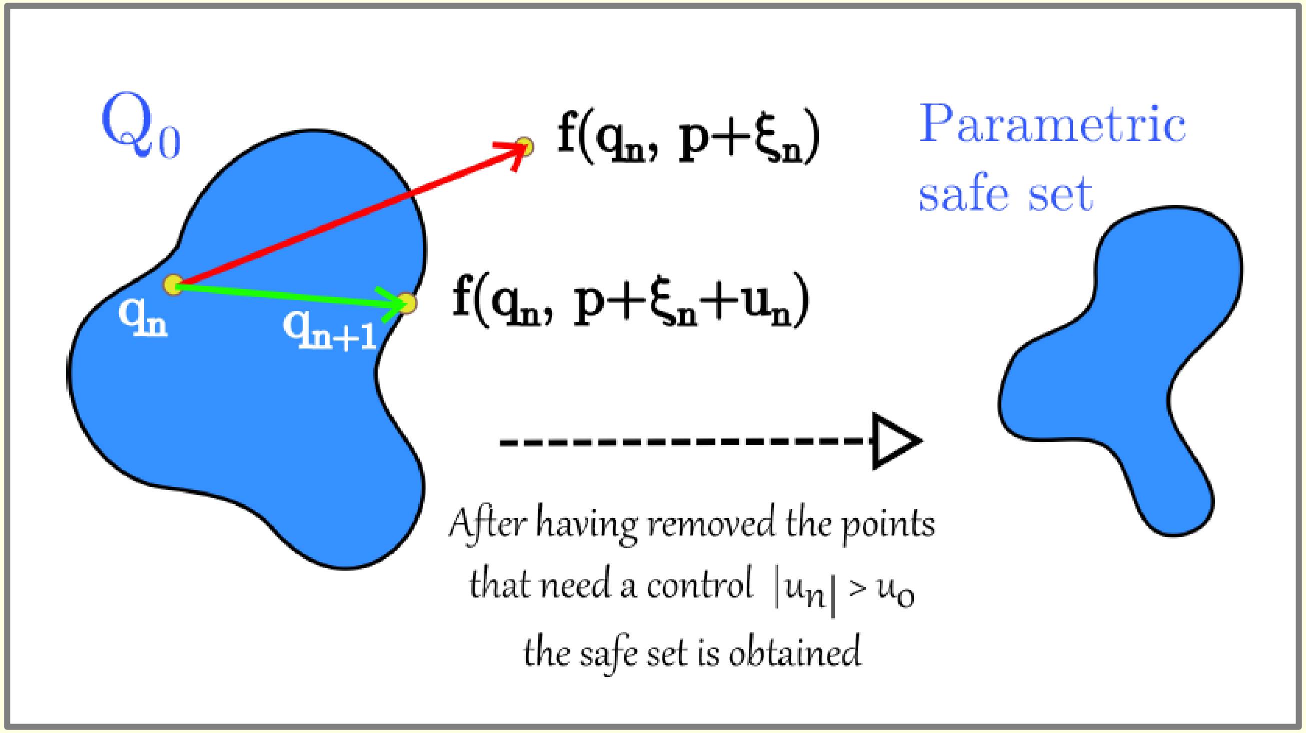
Parametric partial control

$$q_{n+1} = f(q_n, p + \xi_n + u_n)$$

$|\xi_n| \leq \xi_0$

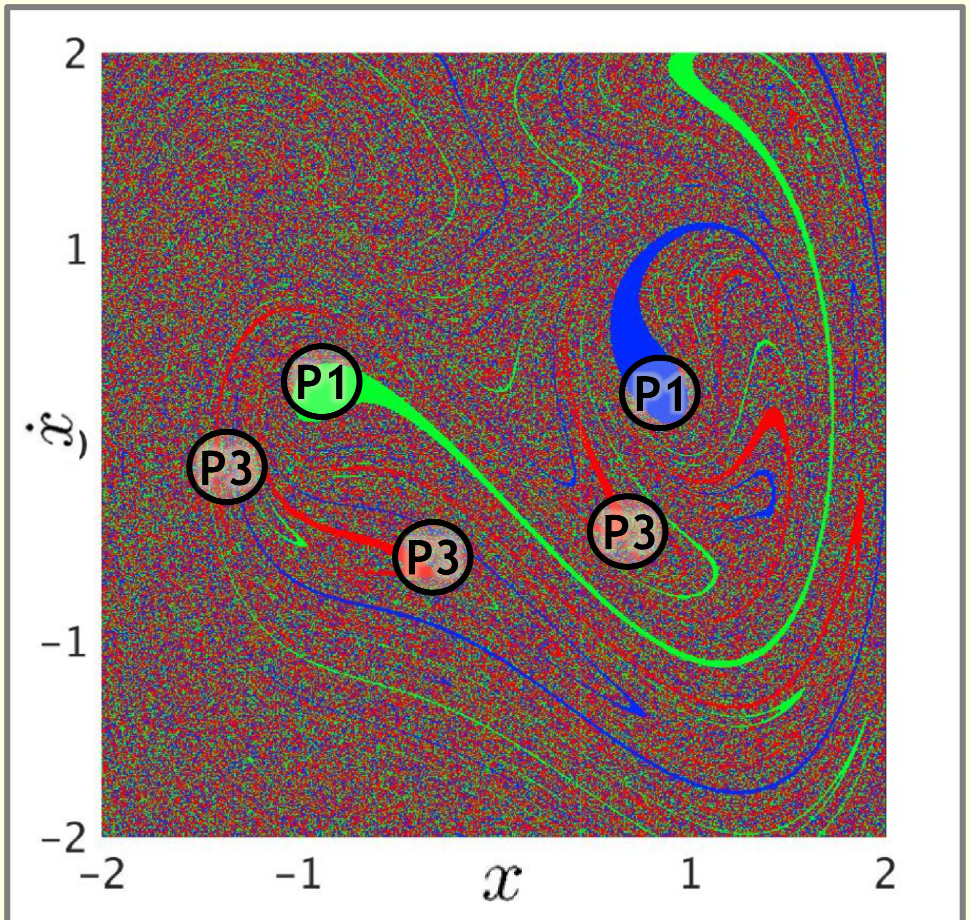
$|u_n| \leq u_0$

$u_0 < \xi_0$

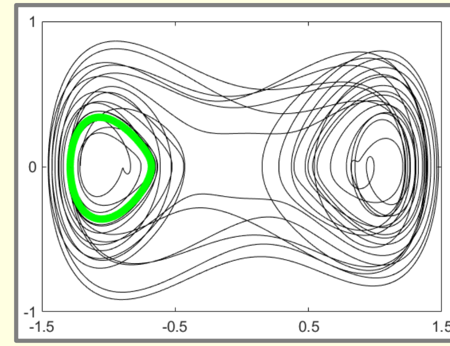


Parametric partial control in the Duffing oscillator

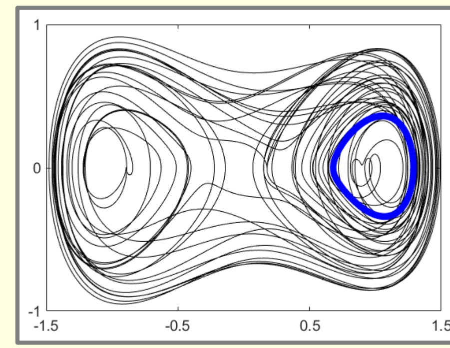
$$\ddot{x} + 0.15\dot{x} - x + x^3 = \boxed{0.245} sin(t)$$



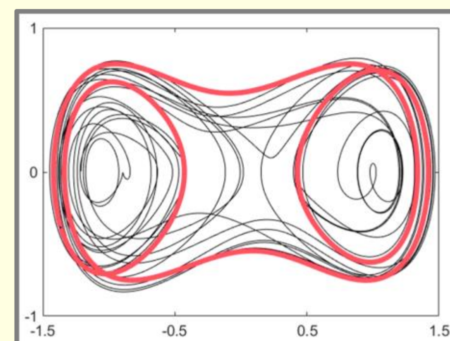
P1



P1



P3



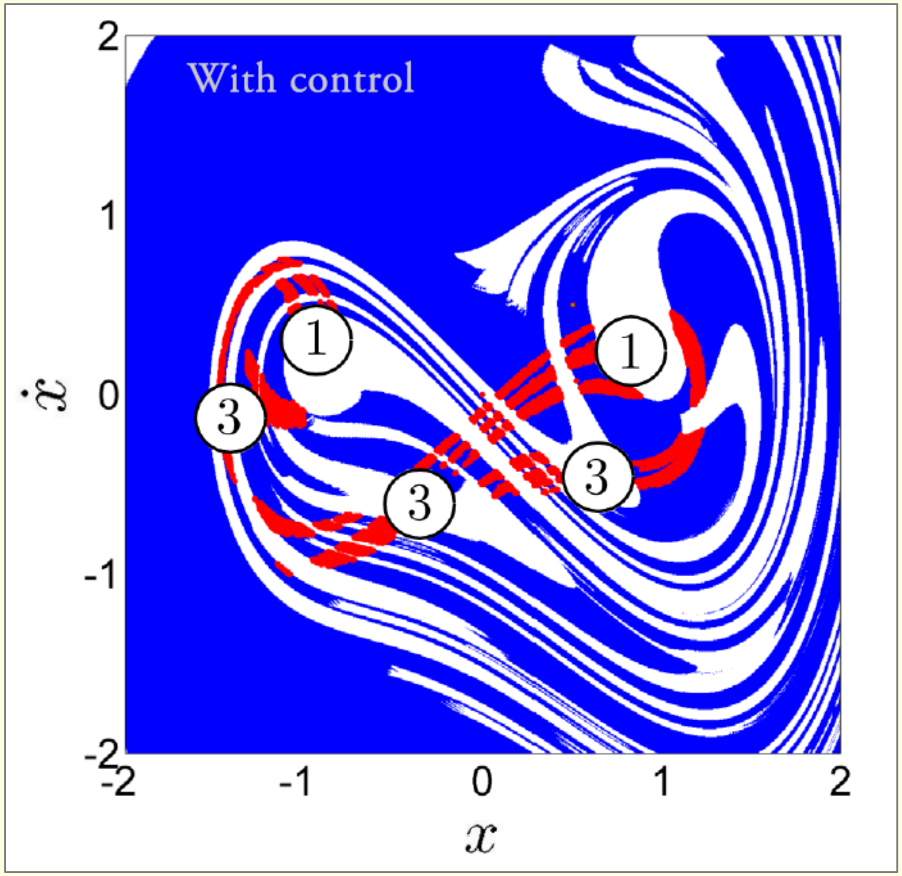
Parametric partial control in the Duffing oscillator

$$\ddot{x} + 0.15\dot{x} - x + x^3 = \boxed{0.245}\,sin(t)$$

$$\boxed{0.245 + \xi_n + u_n}$$

Goal:

Avoid the periodic attractors



Safe set

$$\boxed{\xi_0 = 0.020}$$

$$\boxed{u_0 = 0.014}$$

7- Controlling time-delay coordinate maps

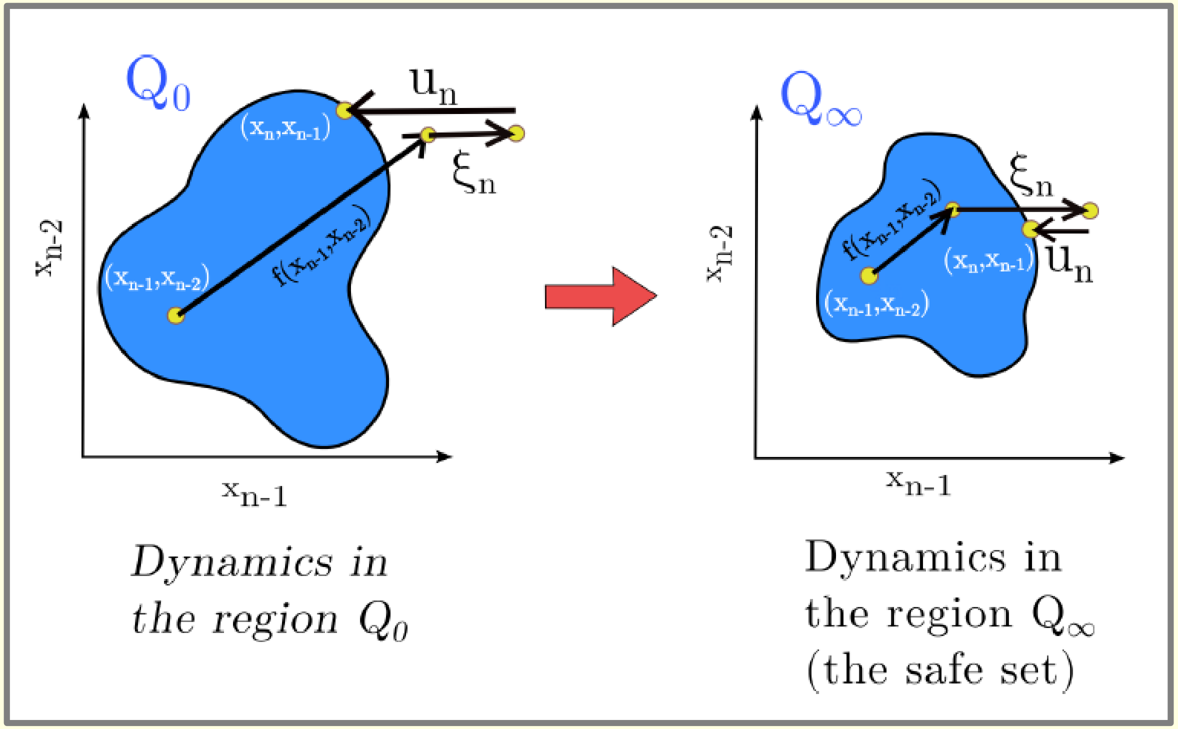
R. Capeáns, J. Sabuco and M. A. F. Sanjuán. Partial control of delay-coordinate maps. *Nonlinear Dynamics* **92**, 1419-1429, (2018).

Time-delay coordinate maps: partial control scheme

$$\boxed{x_n = f(x_{n-1}, x_{n-2}, \ldots) + \xi_n + u_n}$$

$$\boxed{|\xi_n| \leq \xi_0}$$

$$\boxed{|u_n| \leq u_0}$$



Only the present state can be controlled

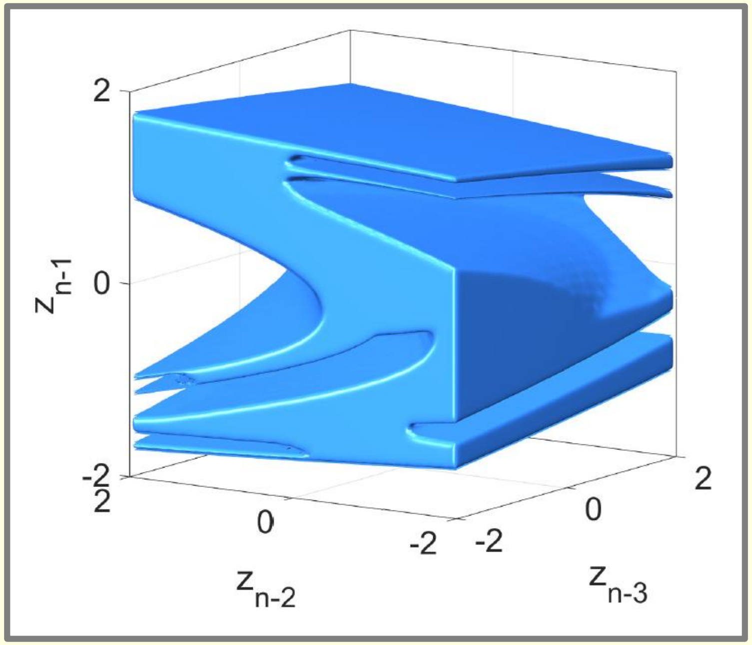
Time-delay coordinate maps: The 3D Hénon map

$$\begin{aligned} x_n &= b z_{n-1} \\ y_n &= c x_{n-1} + b z_{n-1} \\ z_n &= 1 + y_{n-1} - a z_{n-1}^2 \end{aligned} \quad \longrightarrow \quad z_n = 1 - a z_{n-1}^2 + b z_{n-2} - c b z_{n-3} + \xi_n + u_n$$

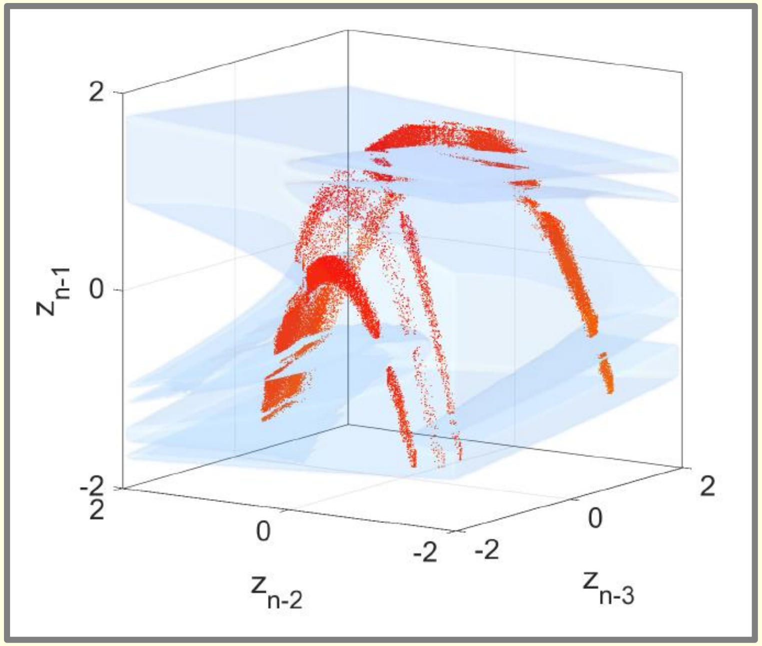
$a = 1.1,\ b = 0.3,\ c = 1$

Goal: keep the trajectory in the box

The safe set $\xi_0 = 0.12$ $u_0 = 0.08$



Controlled trajectory



8- A new approach: the safety function

R. Capeáns, J. Sabuco and M. A. F. Sanjuán. A new approach of the partial control method in chaotic systems. Nonlinear Dynamics (2019)
https://arxiv.org/abs/1902.06238

The safety function

$$q_{n+1} = f(q_n, \xi_n) + u_n$$

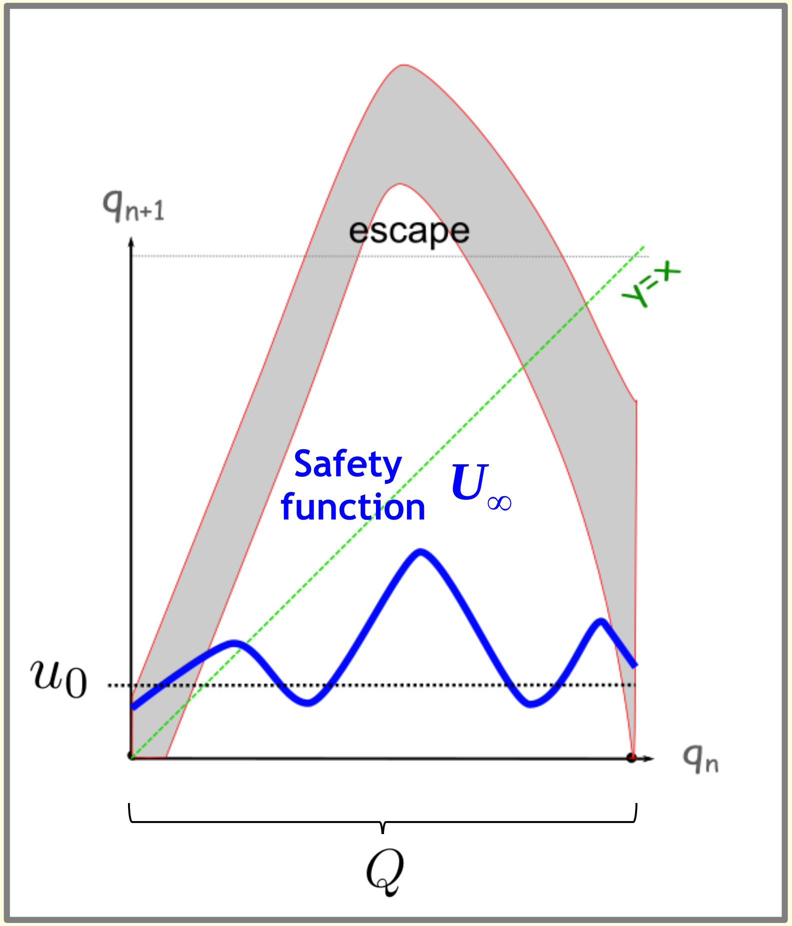
Disturbance *Control*

$|\xi_n(q)| \leq \xi_0(q)$ $\qquad$ $|u_n| \leq u_0$ (Set at the end)

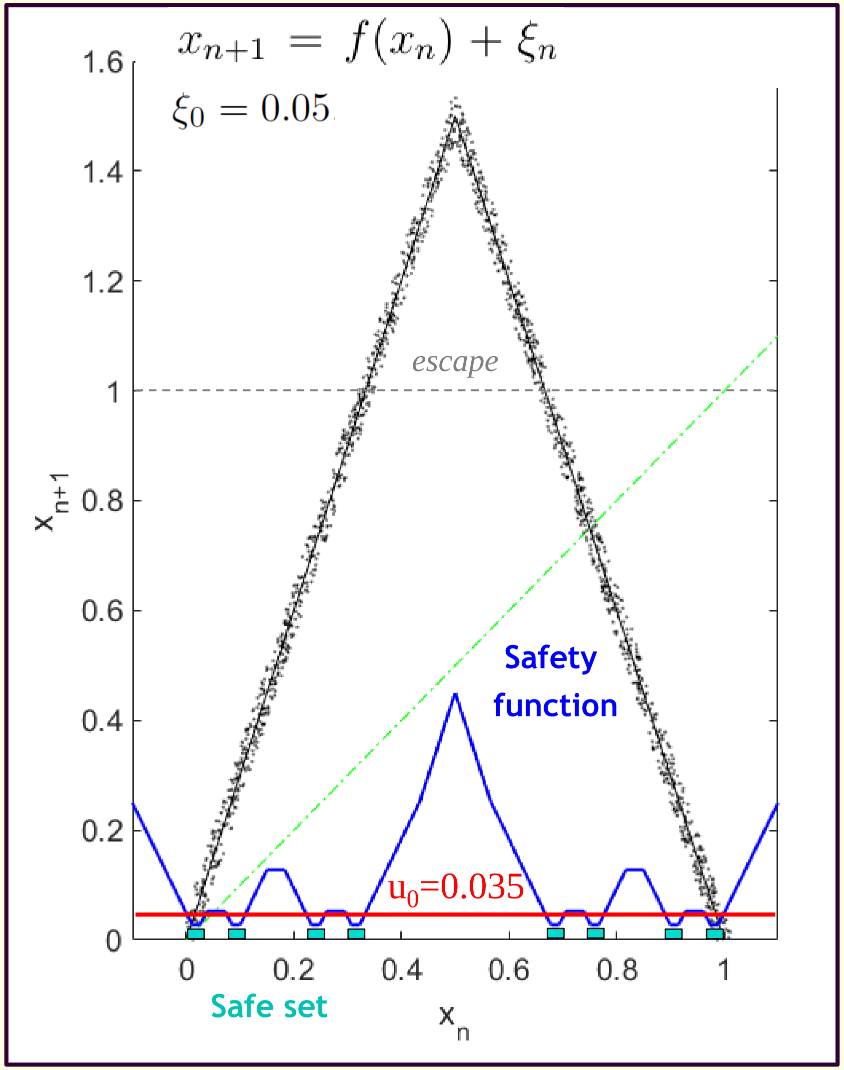
The safety function

$U_\infty(q)$= **minimum upper bound to keep the point *q* in the region Q, forever.**

Recursive algorithm

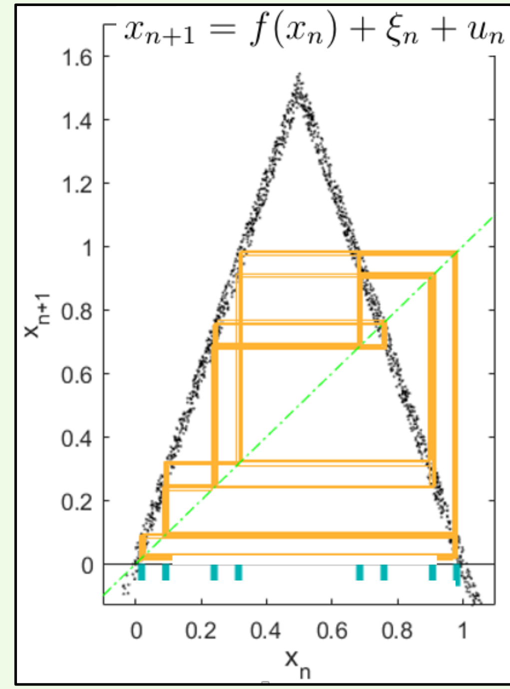


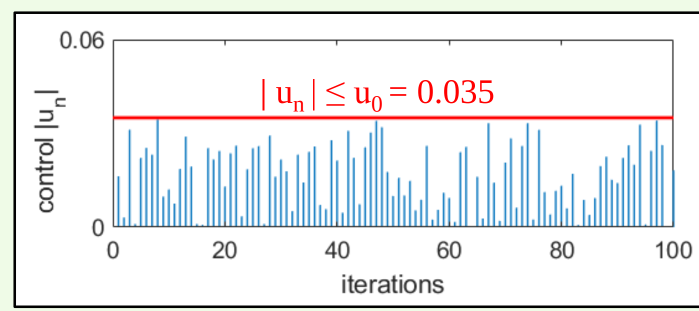
The safety function and the safe set



Q=[-0.1, 1.1]

Controlled trajectory



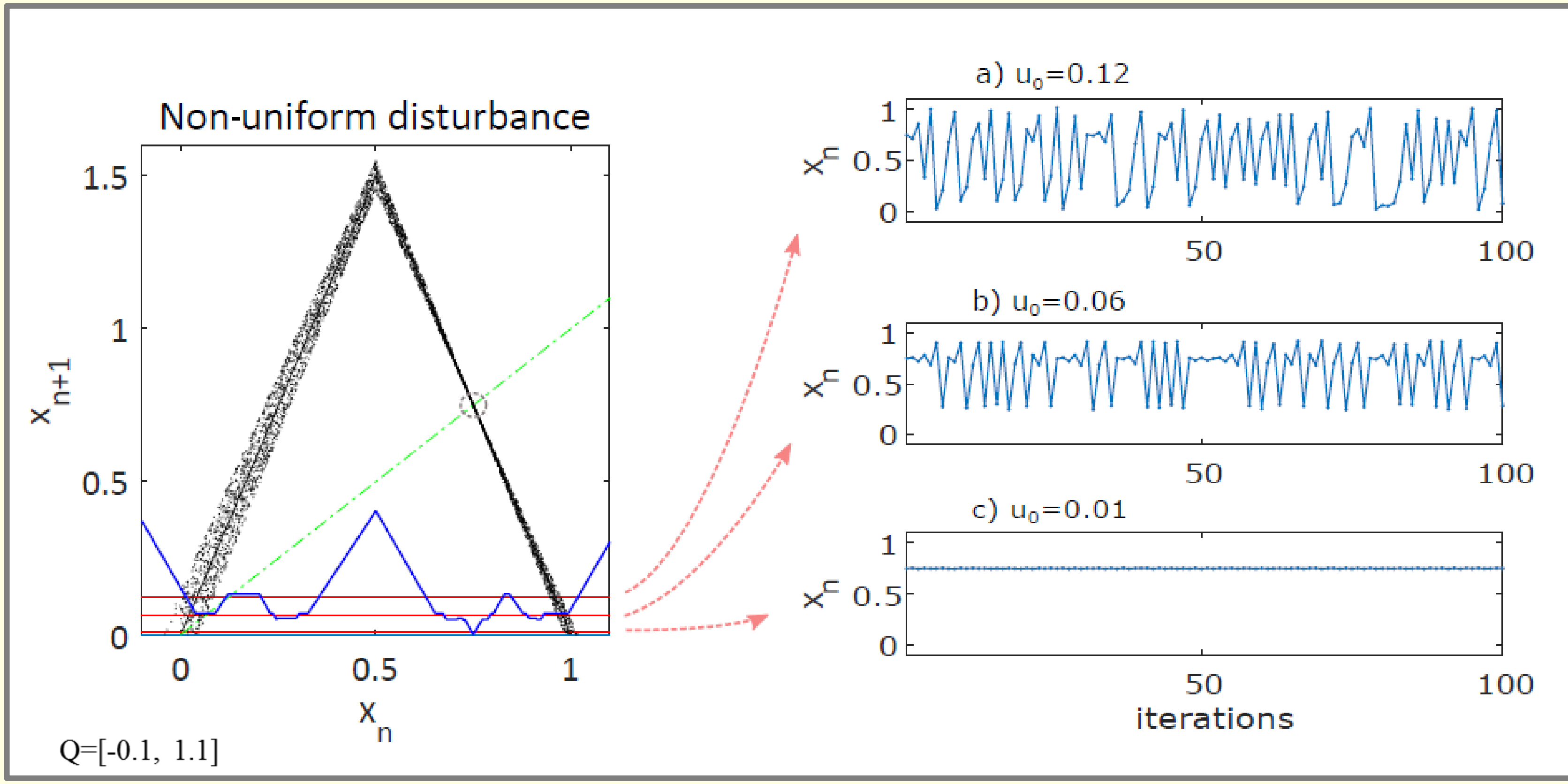


The safety function with non-uniform disturbance

The slope-3 tent map

Non-uniform disturbance

$$x_{n+1} = \begin{cases} 3x_n + \boxed{\xi_n(4x_n - 3)} + u_n & \text{for } x_n \leq \frac{1}{2} \\ 3(1 - x_n) + \boxed{\xi_n(4x_n - 3)} + u_n & \text{for } x_n > \frac{1}{2}, \end{cases}$$

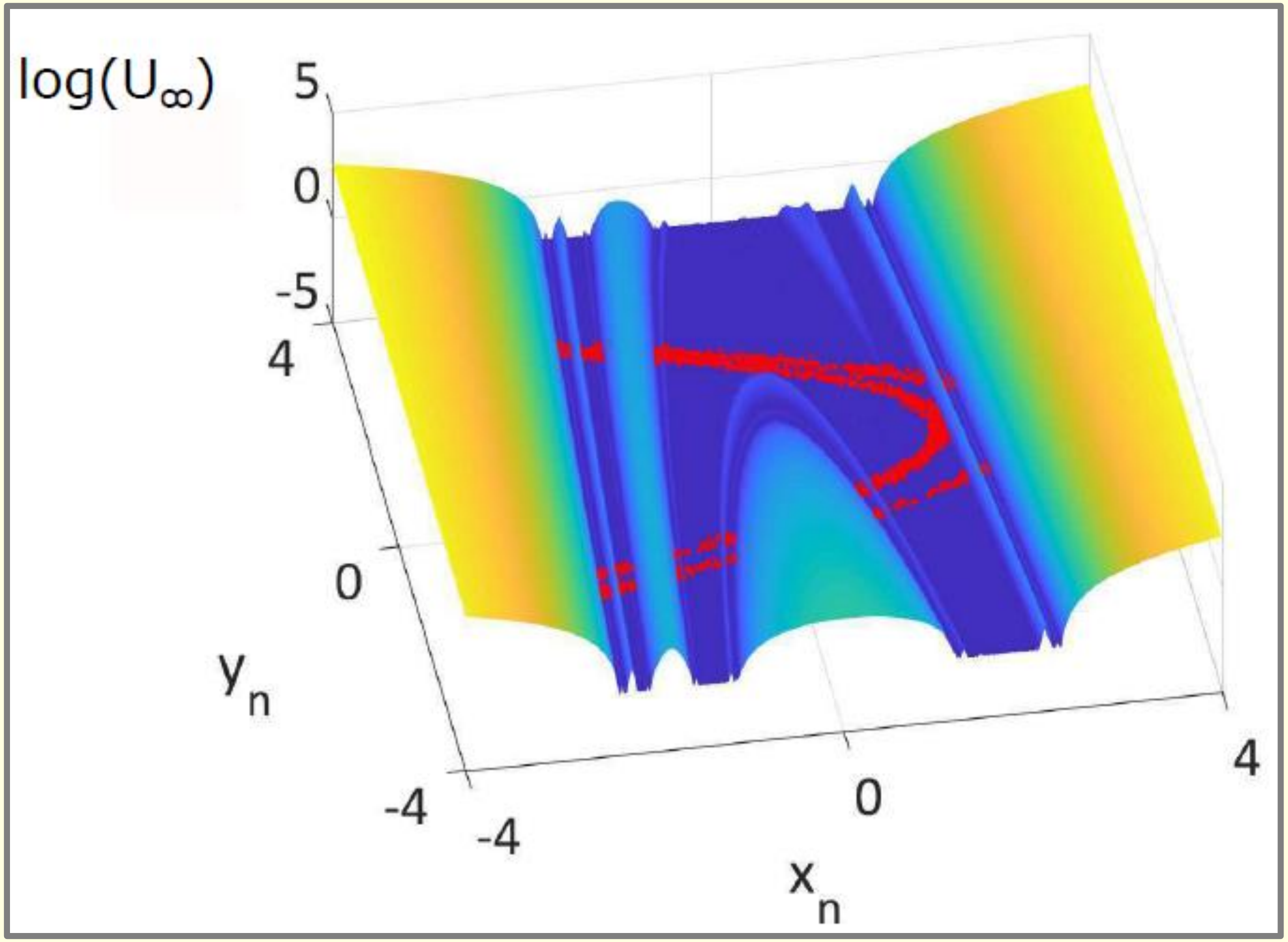


The safety function in the Hénon map

$$x_{n+1} = a - by_n - x_n^2 + \xi_n^x + u_n^x$$
$$y_{n+1} = x_n + \xi_n^y + u_n^y.$$

$a = 2.16$ and $b = 0.3$

$Q_0 = [-4, 4] \times [-4, 4]$



$\xi_0 = 0.1$

$u_0 = 0.08$

8- Conclusions

Introduction and first applications:

- Partial control is applied on dynamical systems that exhibits transient chaos, with the aim of avoiding the crisis even in presence of noise.
- This method is able to keep the amount of control smaller than the disturbance affecting the system
- An ecological model was considered with the aim of avoiding the extinction of one species. By computing a one-dimensional safe set the crisis was suppressed.
- This control method was also applied to the Lorenz system by using a 1D, 2D and 3D maps, obtaining for the first time a three-dimensional safe set.

The partial control technique was adapted to be applied in different scenarios:

- First, we have shown that safe sets can be used in a dual way, to avoid the escape or to accelerate it.
- Secondly, we considered the scenario where a parameter of the system is affected by the disturbance. We found that parametric safe sets exist when control is applied on the parameter of the system.
- Finally, the family of time-delay coordinates maps was studied. We show that it is possible to control the orbits with the only observation and control of one variable.

Conclusions

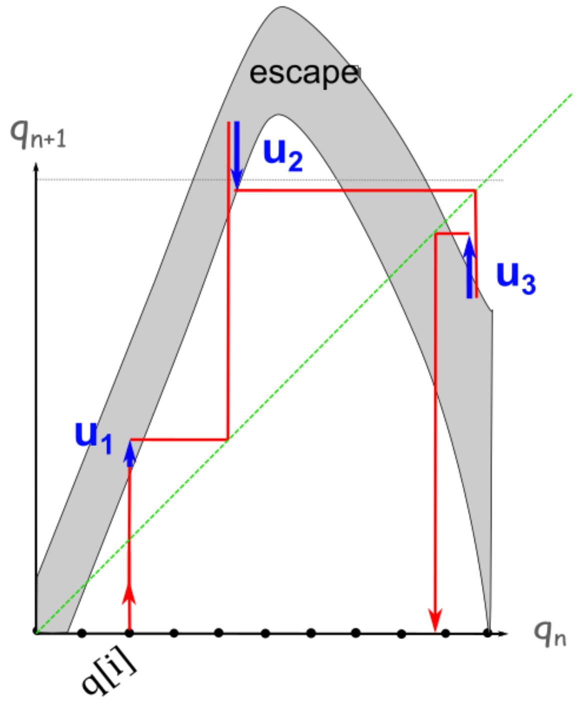
A new approach:

- With the same philosophy of partial control, we introduce the concept of the safety function, which is a generalization of the safe set.
- This function indicates the minimum upper bound of control necessary to keep each initial condition in the region Q forever.
- The use of the safety function allows us to treat more diverse scenarios, being specially useful in case of experimental time series.

Publications

- R. Capeáns, J. Sabuco and M. A. F. Sanjuán. A new approach of the partial control method in chaotic systems. Nonlinear Dynamics (2019) https://arxiv.org/abs/1902.06238
- R. Capeáns, J. Sabuco and M.A.F.Sanjuán. Partial control of chaos: how to avoid undesirable behaviors with small controls in presence of noise. *Discrete and Continuous Dynamical Systems - Series B* **2,** 3237-3274, (2018).
- R. Capeáns, J. Sabuco and M. A. F. Sanjuán. Partial control of delay-coordinate maps. *Nonlinear Dynamics* **92**, 1419-1429, (2018).
- R. Capeáns, J. Sabuco and M. A. F. Sanjuán. Escaping from a chaotic saddle in the presence of noise. *International Journal of Nonlinear Dynamics and Control* **1**, 1-8, (2017).
- R. Capeáns, J. Sabuco, M. A. F. Sanjuán and J. A. Yorke. Partially controlling transient chaos in the Lorenz equations. *Philosophical Transactions of the Royal Society A* **375**, 2088, (2017).
- R. Capeáns, J. Sabuco and M. A. F. Sanjuán. Parametric partial control of chaotic systems. *Nonlinear Dynamics* **2**, 869-876, (2016).
- R. Capeáns, J. Sabuco and M. A. F. Sanjuán. When less is more: Partial control to avoid extinction of predators in an ecological model. *Ecological Complexity* **19**, 1-8, (2014).

Meaning and computation of the safety function



Upper bound =max(|u_1|,|u_2|,|u_3|)

(For q[i] and k=3 iterations)

$U_k[i]$= minimum upper bound to keep the point q[i] in the region Q, during k iterations

Is it possible to find the function U_k ?

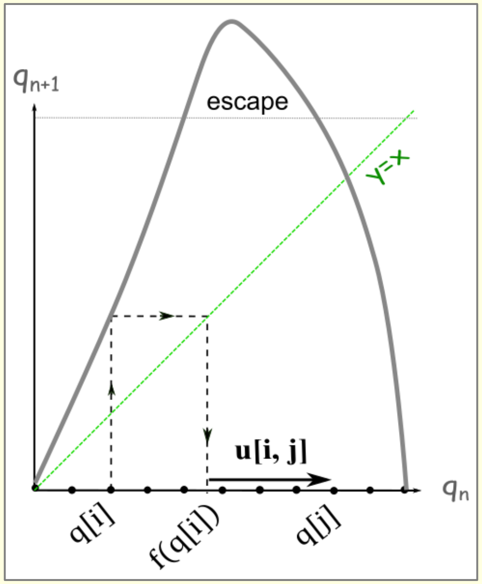
The function U_k (in absence of disturbances)

What is the minimum control bound for every point $q \in Q$ to remain in Q during k iterations?

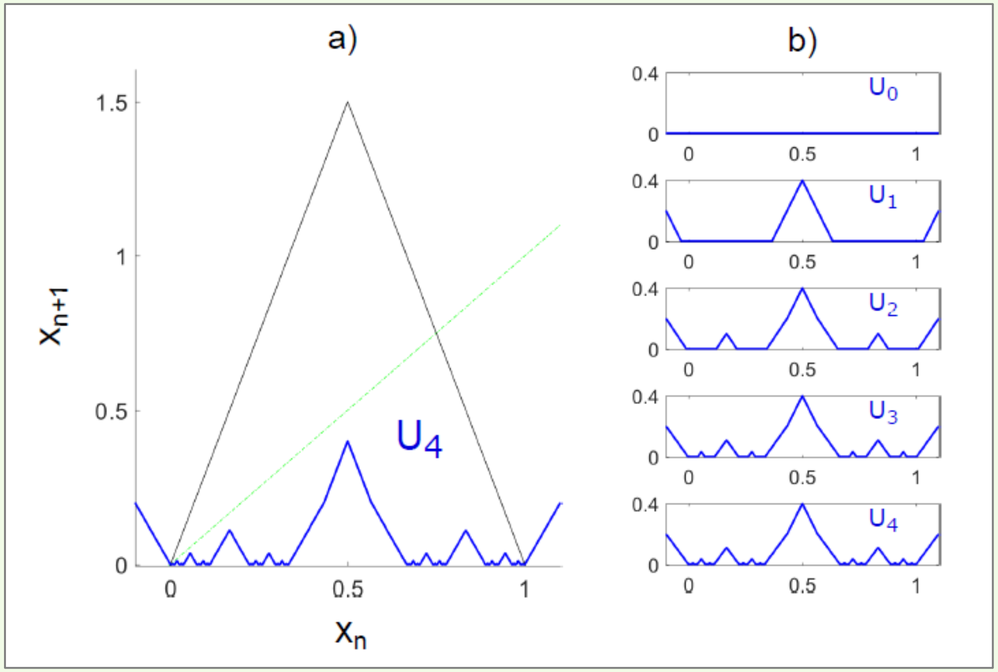
Discretization

$$q_{n+1} = f(q_n) + u_n$$

$$q[j] = f(q[i]) + u[i,j]$$



$$U_{k+1}[i] = \min_{1\le j\le N} \left(\max\left(u[i,j],\, U_k[j] \right) \right)$$



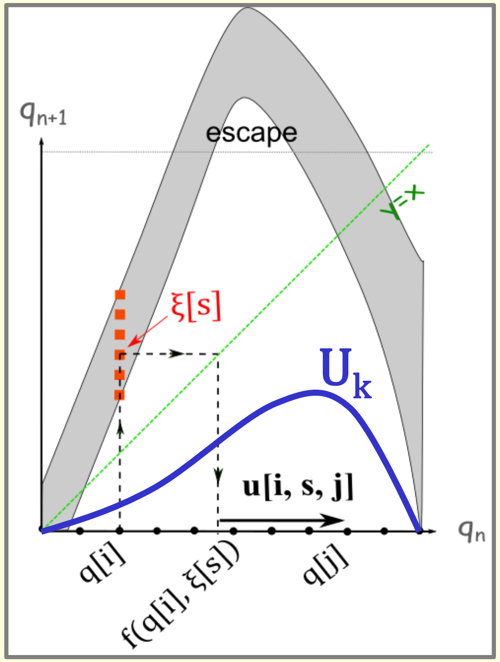
Meaning and computation of the safety function

Discretization

$$q_{n+1} = f(q_n, \xi_n) + u_n$$

↓

$$q[j] = f\big(q[i], \xi[s]\big) + u[i, s, j]$$



Recursive algorithm

$$U_{k+1}[i] = \max_{1\le s\le M_i}\Big(\min_{1\le j\le N}\big(\max\big(u[i,s,j],\, U_k[j]\,\big)\big)\Big)$$

To remain in Q forever we need to find U_∞

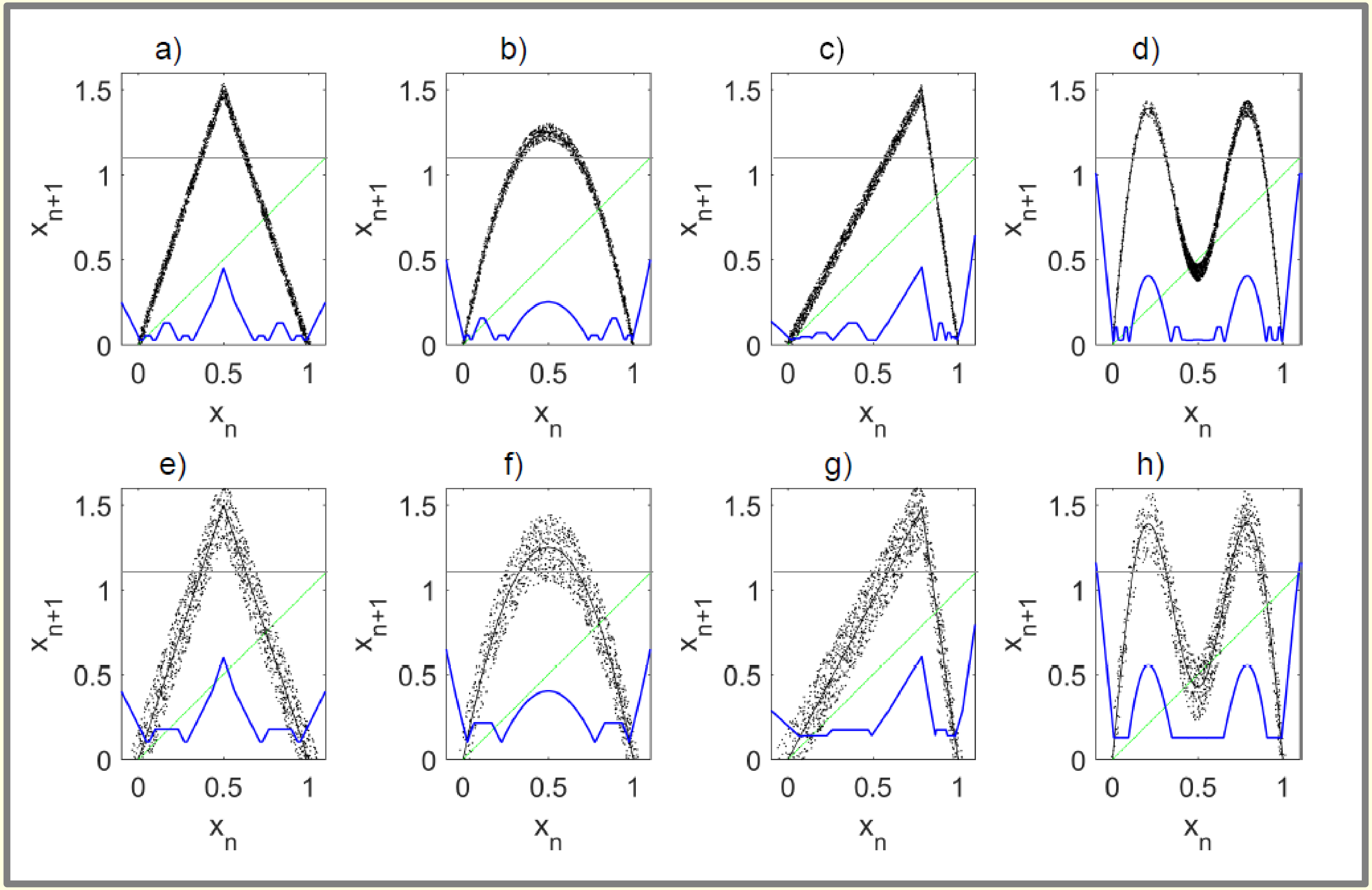
If the algorithm converges... $U_{k+1} = U_k$

and then... $U_\infty = U_k$

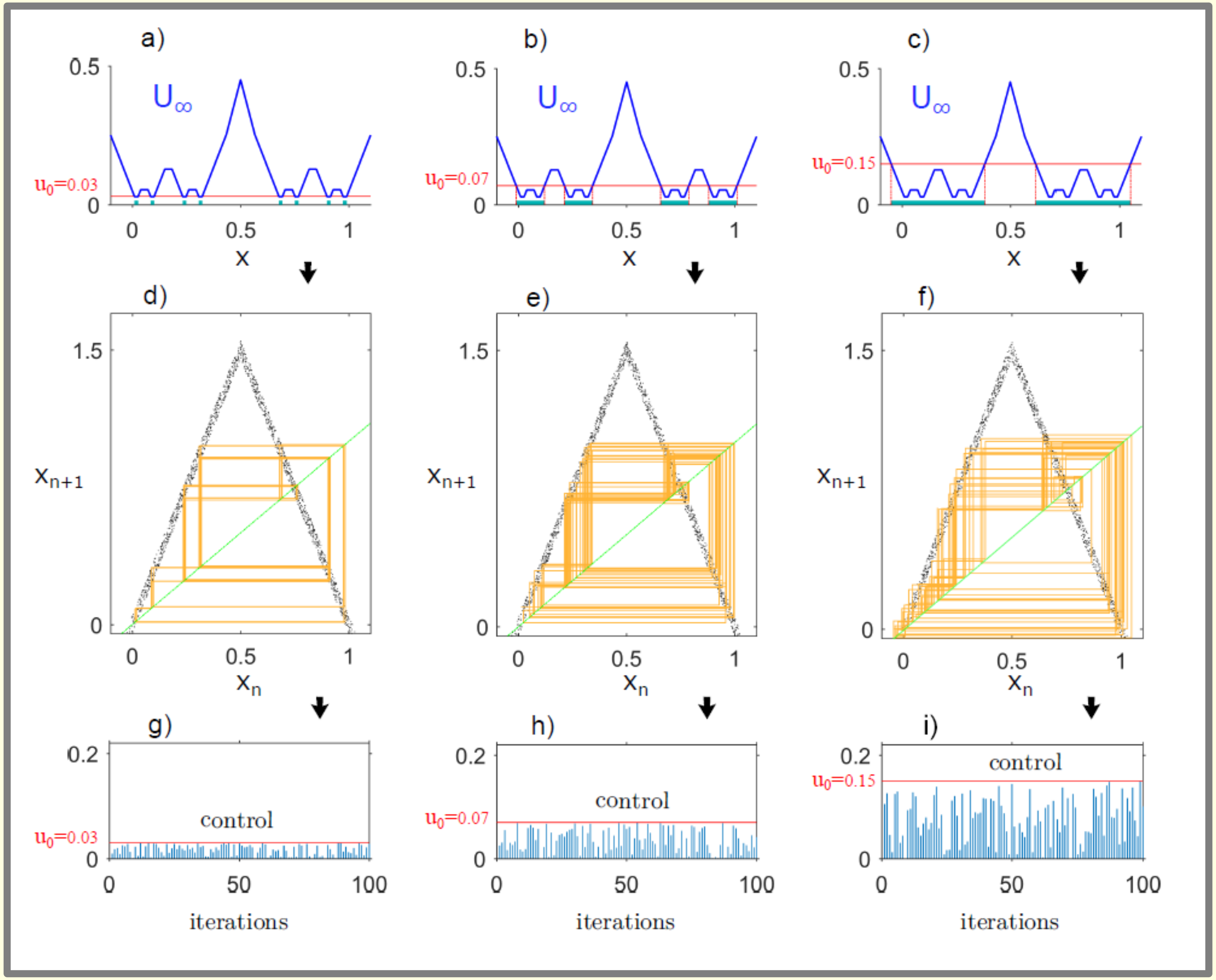
↓

The safety function

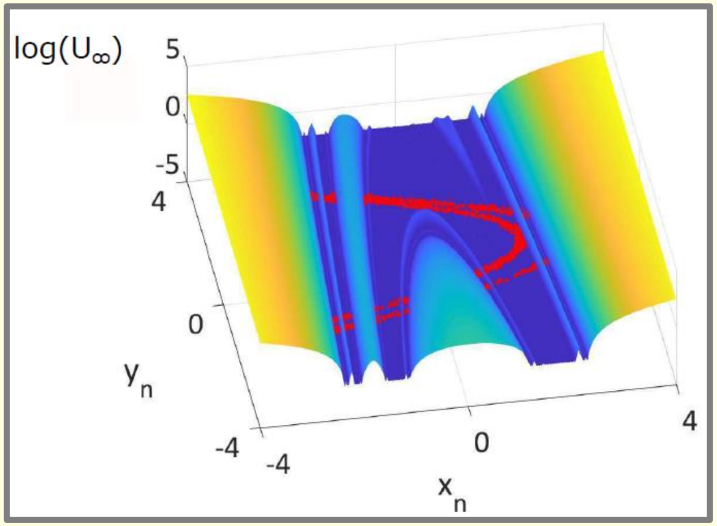
Different safety functions



Recovering the safe set from the safety function



The safety functions in different scenarios



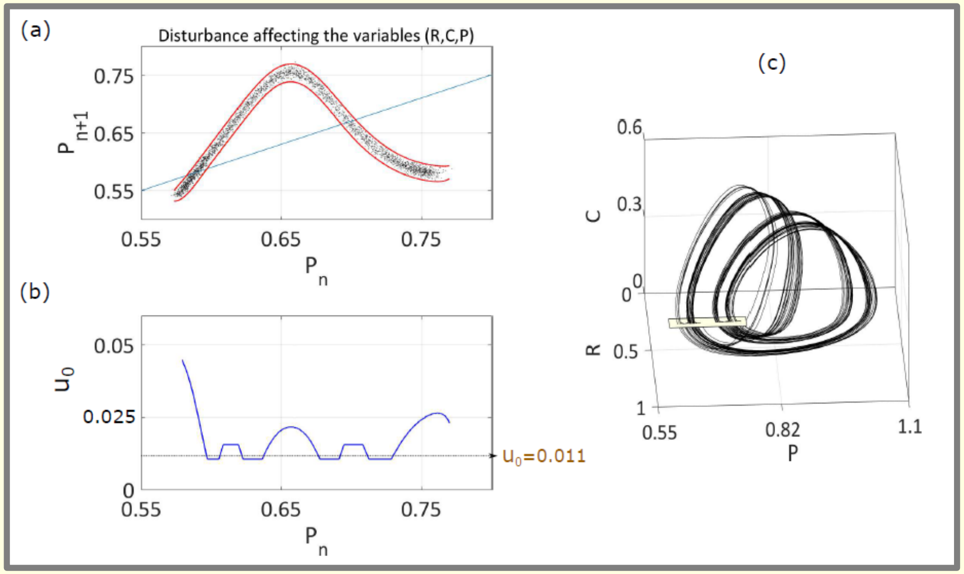
The Hénon map

$$x_{n+1} = a - by_n - x_n^2 + \xi_n^x + u_n^x$$
$$y_{n+1} = x_n + \xi_n^y + u_n^y.$$

$a = 2.16$ and $b = 0.3$

$\xi_0 = 0.1$

$u_0 = 0.08$

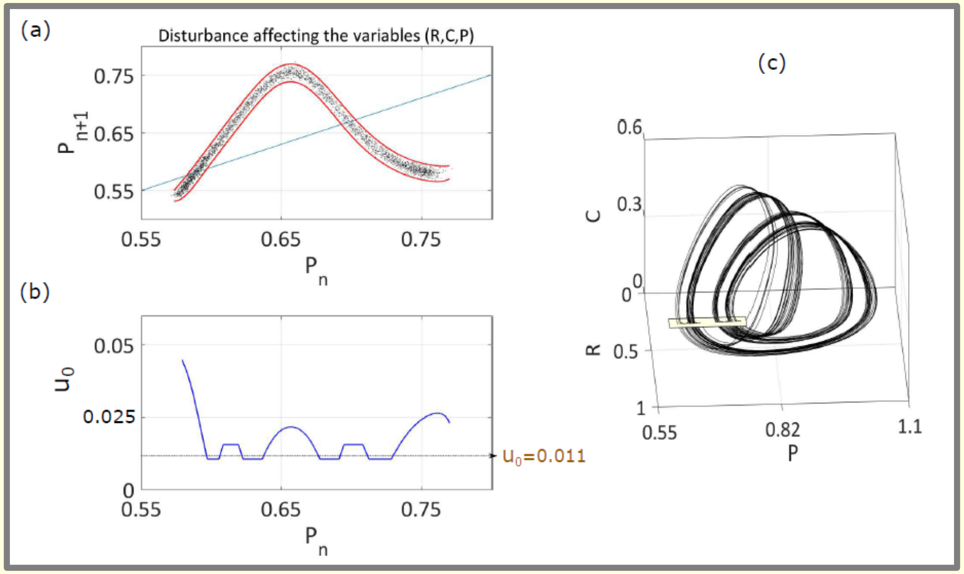


The ecological model:

Continuous noise affecting the variables R,C,P

$$\frac{dR}{dt} = R\left(1 - \frac{R}{K}\right) - \frac{x_c y_c C R}{R + R_0}$$
$$\frac{dC}{dt} = x_c C\left(\frac{y_c R}{R + R_0} - 1\right) - \psi(P)\frac{y_p C}{C + C_0}$$
$$\frac{dP}{dt} = \psi(P)\frac{y_p C}{C + C_0} - x_p P$$

The safety functions in different scenarios



$$\frac{dR}{dt} = R\left(1 - \frac{R}{K}\right) - \frac{x_c y_c C R}{R + R_0}$$
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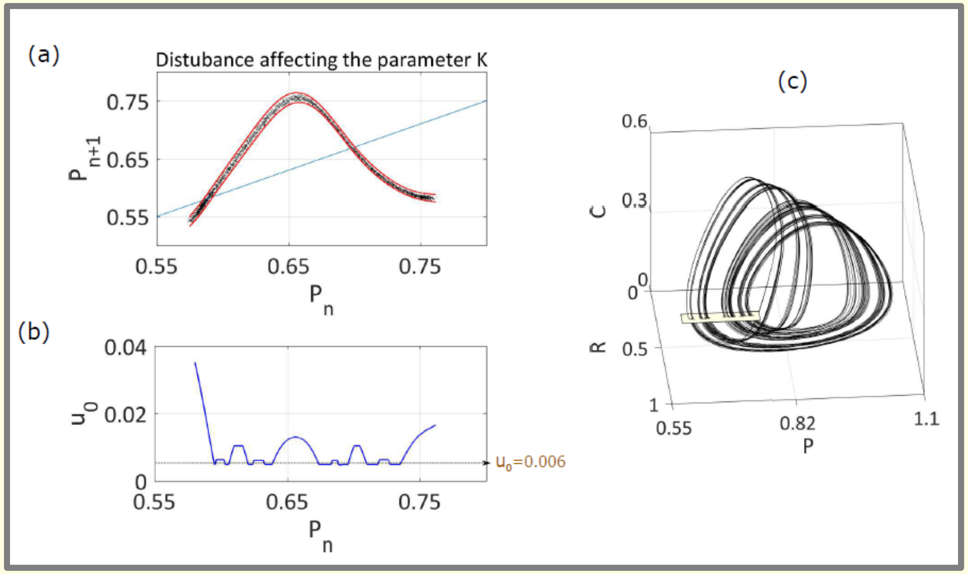
The ecological model:

Continuous noise affecting the variables R,C,P



The ecological model:

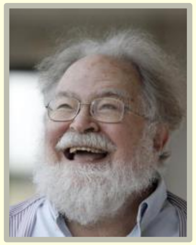
Continuous noise affecting the parameter k

Control of chaos

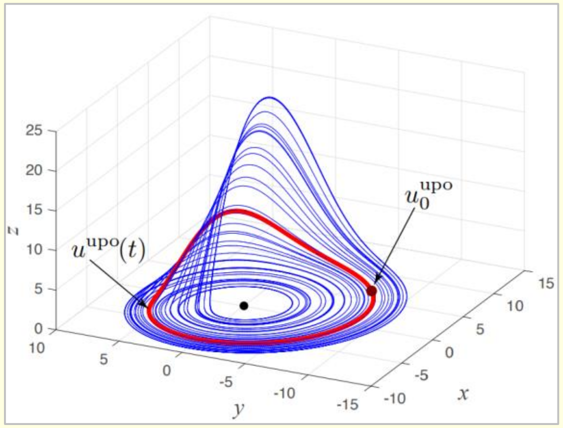
Chaotic motion to regular motion

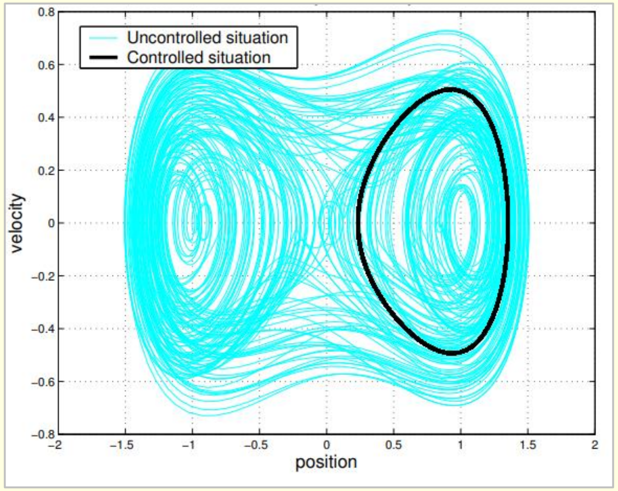
How?

- *OGY method* (1990)



- *Delayed Feedback control* (1992)





Partial control algorithm

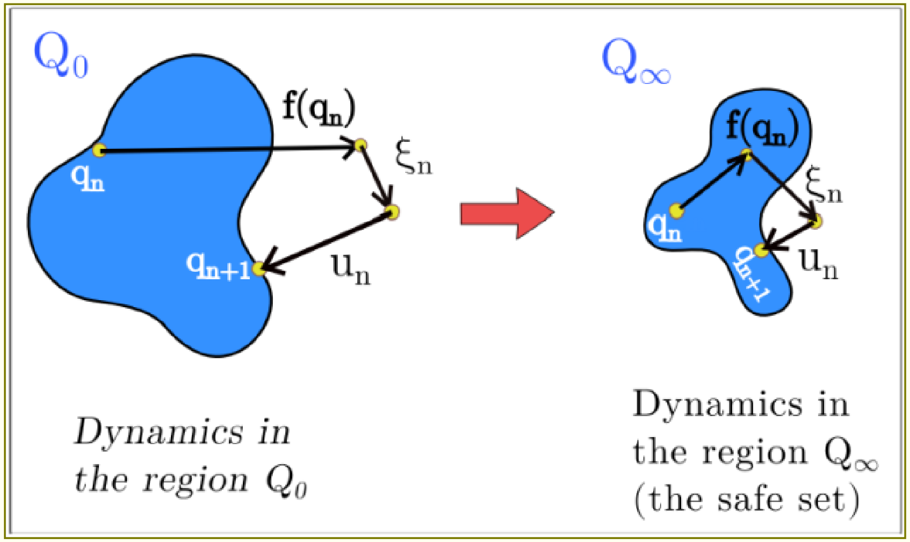
Dynamics in the map

Admissible trajectories

$$\boxed{\mathbf{x}_{n+1} = f(\mathbf{x}_n) + \xi_n + u_n}$$

disturbance *control*

$|\xi_n| \le \xi_0$ $|u_n| \le u_0$



Algorithm to find the safe set.

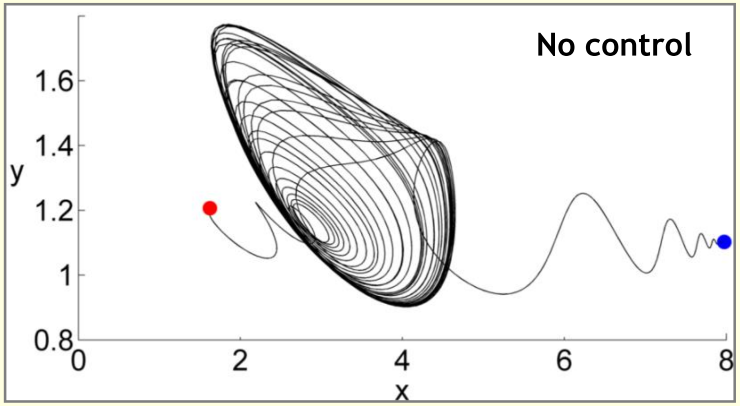
Controlled trajectories in the safe set

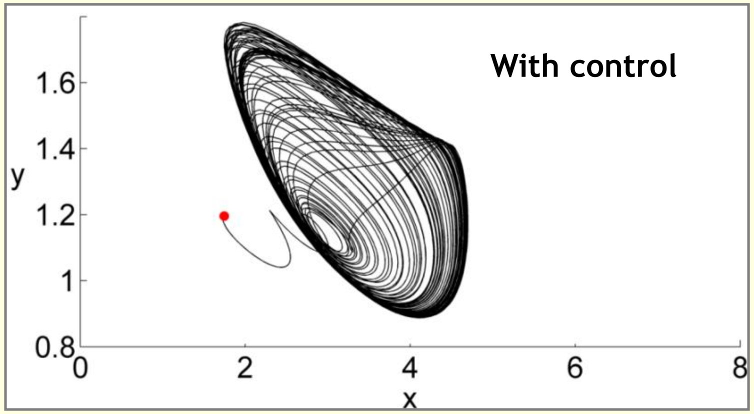
$$\boxed{u_0 < \xi_0}$$

$$\boxed{u_0 < \xi_0}$$

Control of transient chaos

Avoiding the crisis..

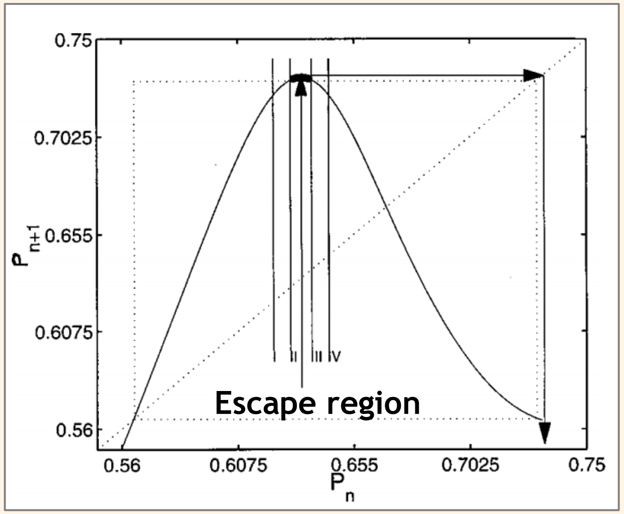




Dhamala and Lai control

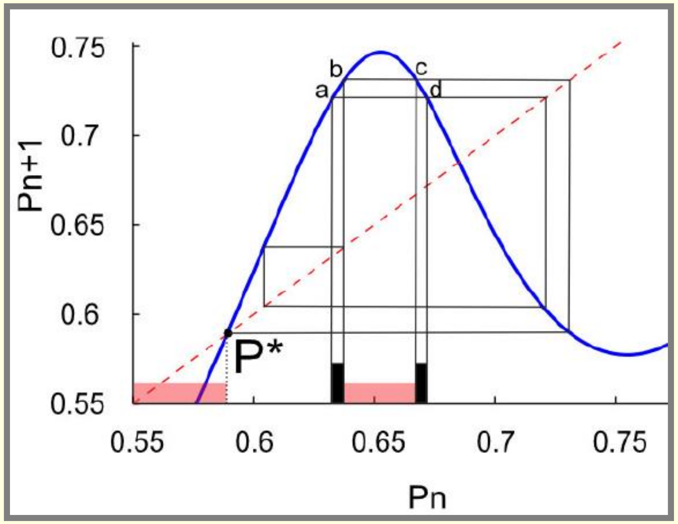
- Identify the **escape regions**
- Apply control to avoid them
- No noise considered !!

M. Dhamala and Y.C. Lai. *Controlling transient chaos in deterministic flows with applications to electrical power systems and ecology*. *Phys. Rev.* E **59**, 1646, 1999



Comparing the control methods

Dhamala-Lai set

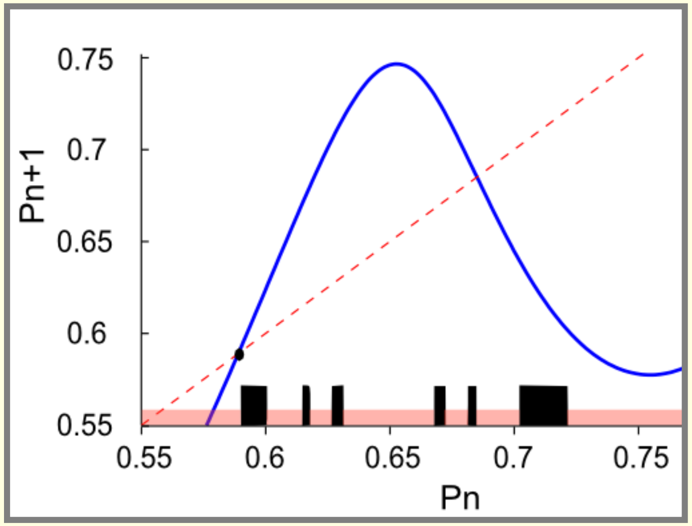


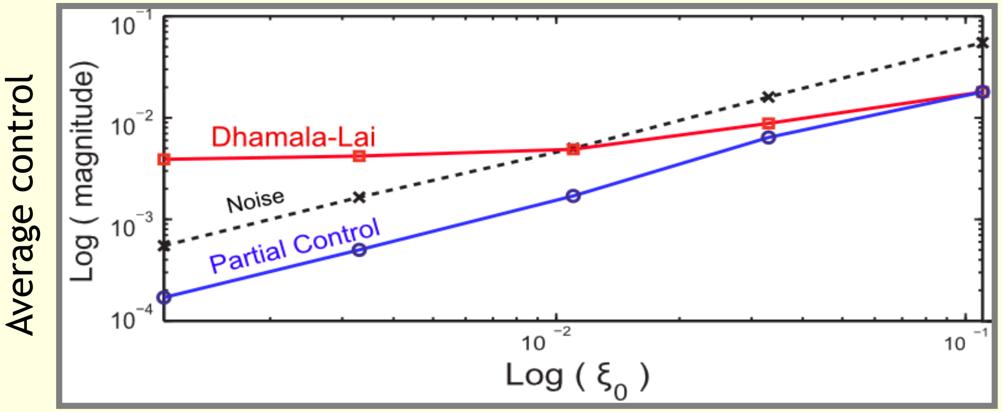
$\xi_0 = 0.010$

$u_0 = 0.007$

Control set

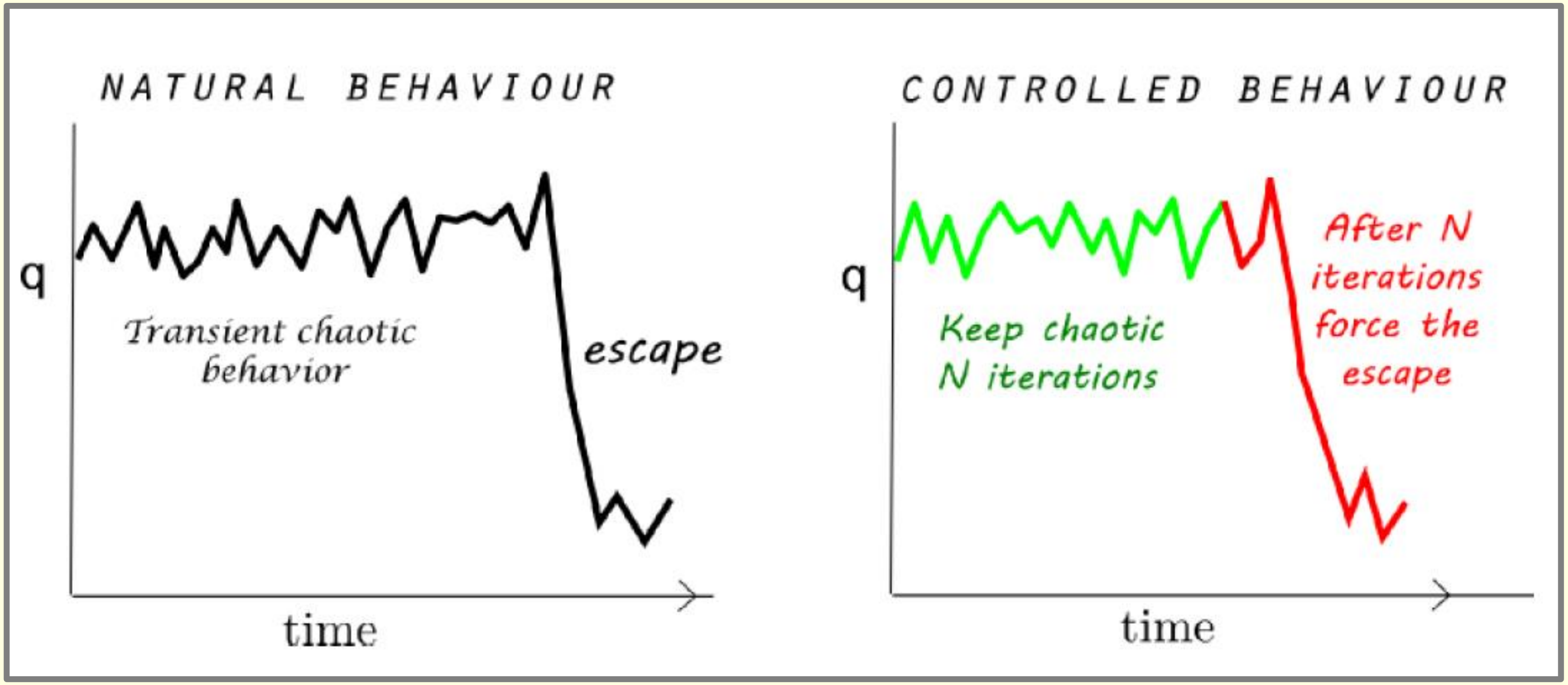
(Partial control) safe set





Escape or not

Based on the idea that the escape time with disturbances resembles the safe set

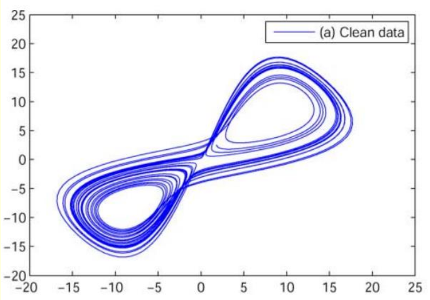


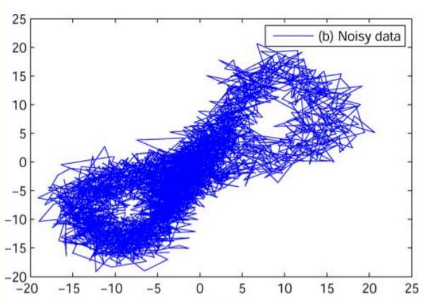
The partial control motivation

Goal: avoid the crisis (escapes) of the transient chaotic trajectory

However:

- Real systems are affected by disturbances
- Source of disturbances: mismatches in model, uncertainty in the exact state of the system, external perturbations..
- Chaotic dynamics is a natural amplifier of noise and therefore control methods may fail.



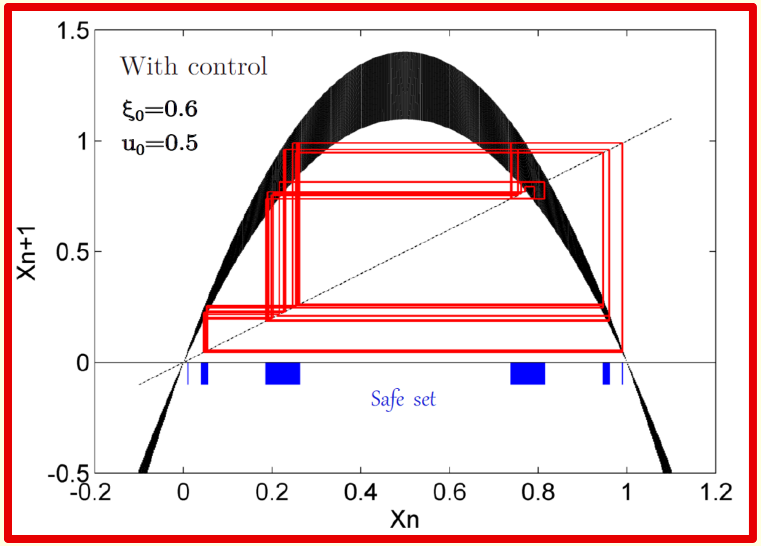


Partial control:

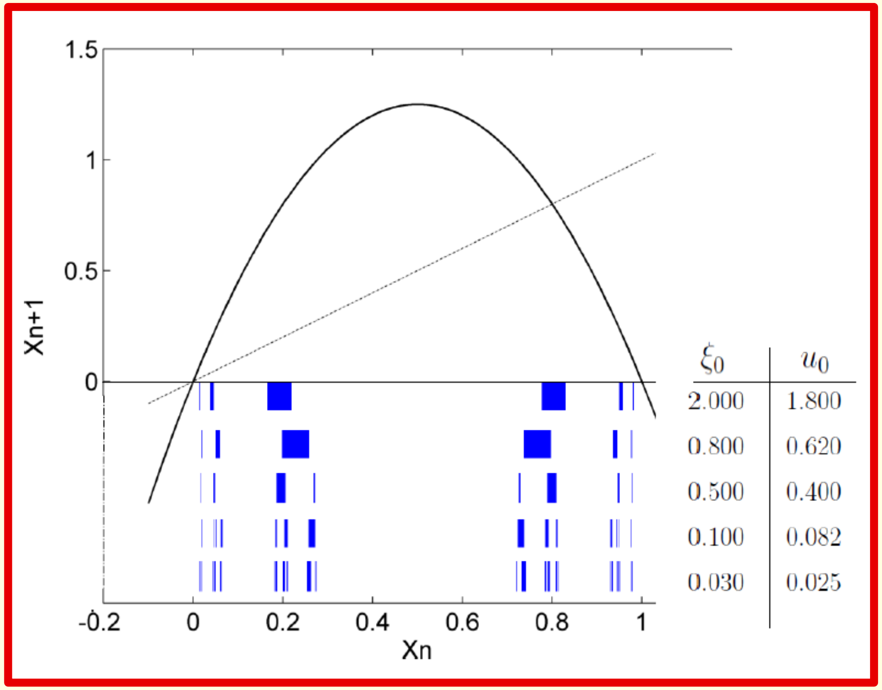
Keep the noise under control and keep the control low

Parametric partial control in the logistic map

$$x_{n+1} = (r + \xi_n + u_n)x_n(1 - x_n)$$



Controlled trajectory using
the parametric safe set



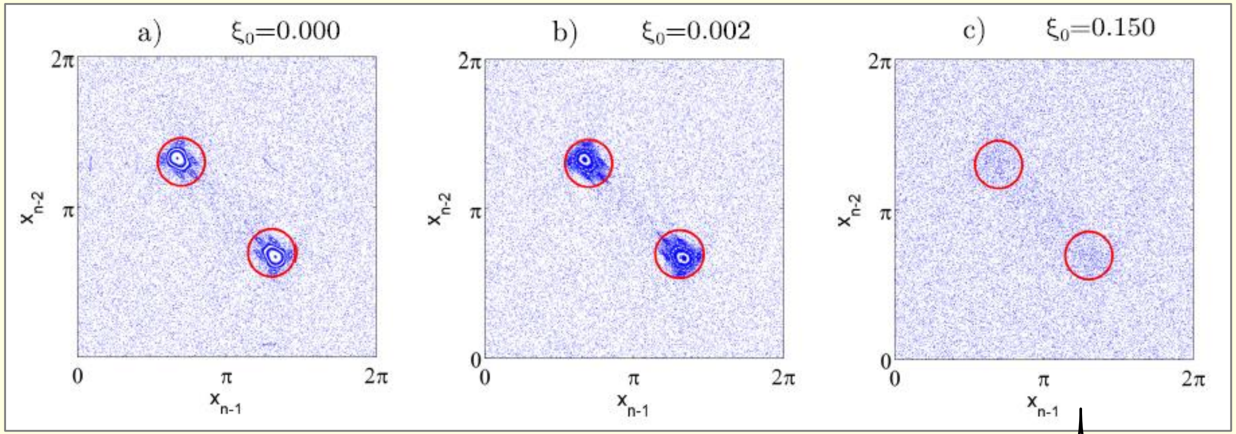
How the parametric safe set changes?

Time-delay coordinate maps: The estandar map

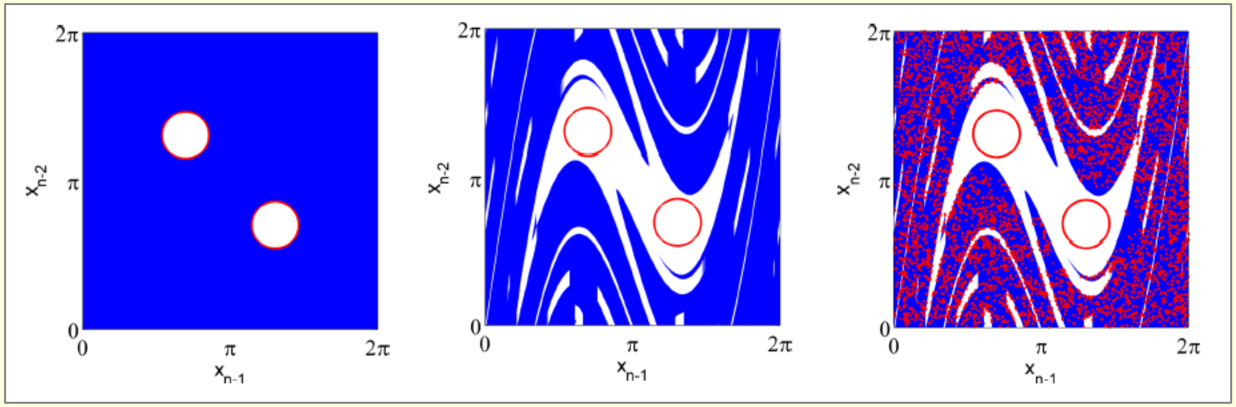
$$y_n = y_{n-1} + K \sin x_{n-1}$$
$$x_n = x_{n-1} + y_n,$$

$\longrightarrow$

$$x_n = 2x_{n-1} - x_{n-2} + K \sin x_{n-1} + \xi_n + u_n$$



Goal:
avoid the KAM islands

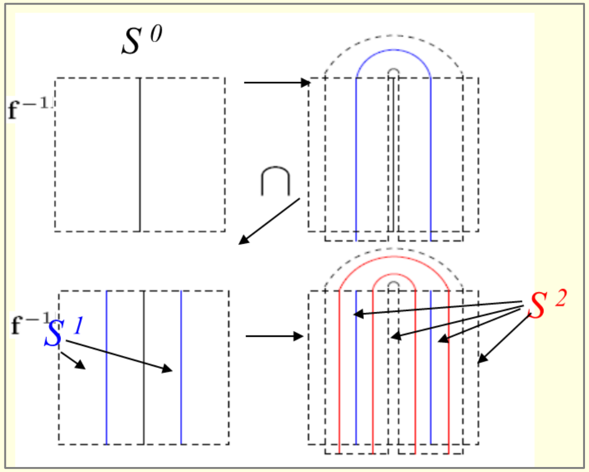


$$\xi_0 = 0.15$$

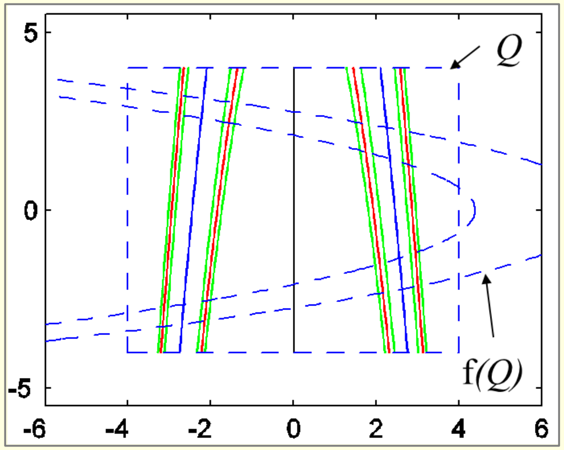
$$u_0 = 0.08$$

Partial control basics

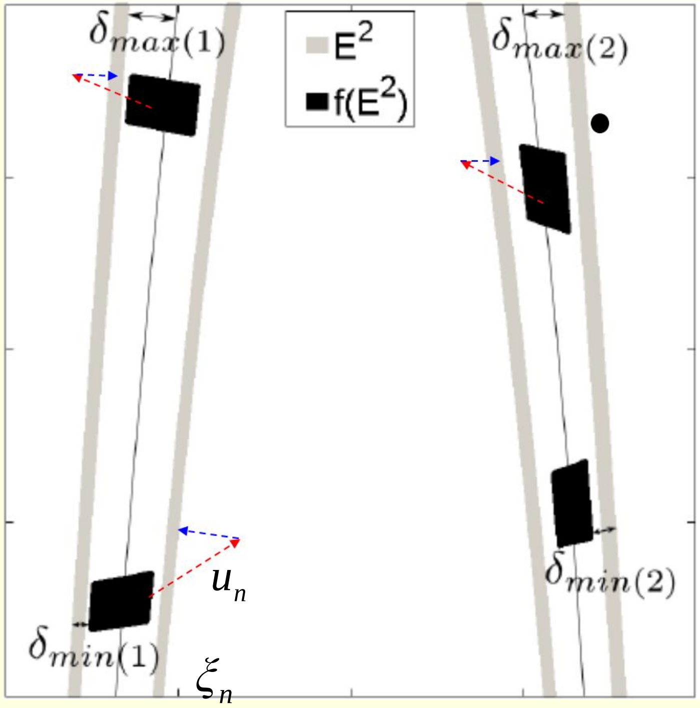
Smale horseshoe



Hénon horseshoe



Partial control in the Hénon map

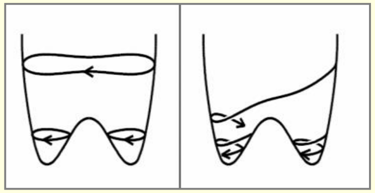


$$\boxed{\mathbf{x}_{n+1} = f(\mathbf{x}_n) + \xi_n + u_n}$$

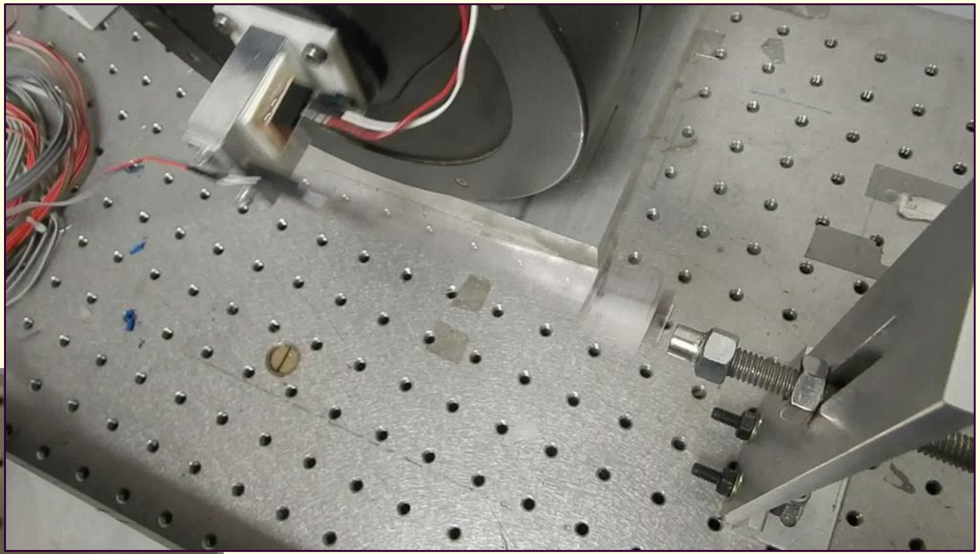
disturbance *control*

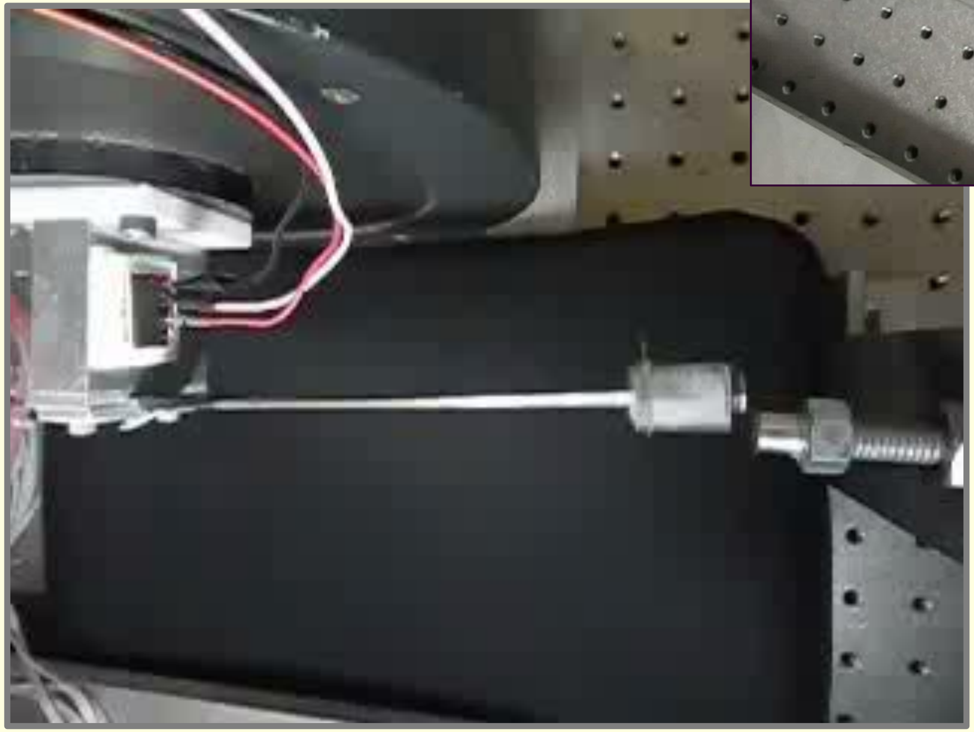
Experimental transient chaos

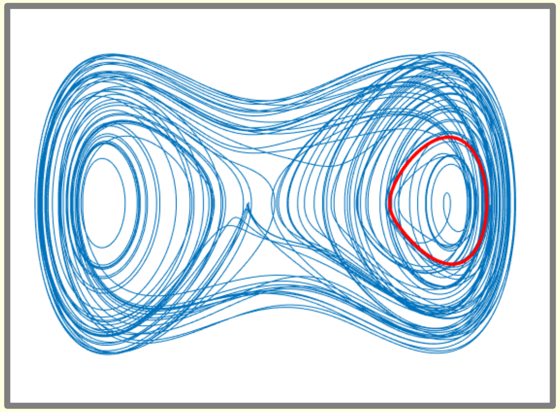
Duffing oscillator



$$\ddot{x} + \delta\dot{x} + \alpha x + \beta x^3 = F cos(wt)$$



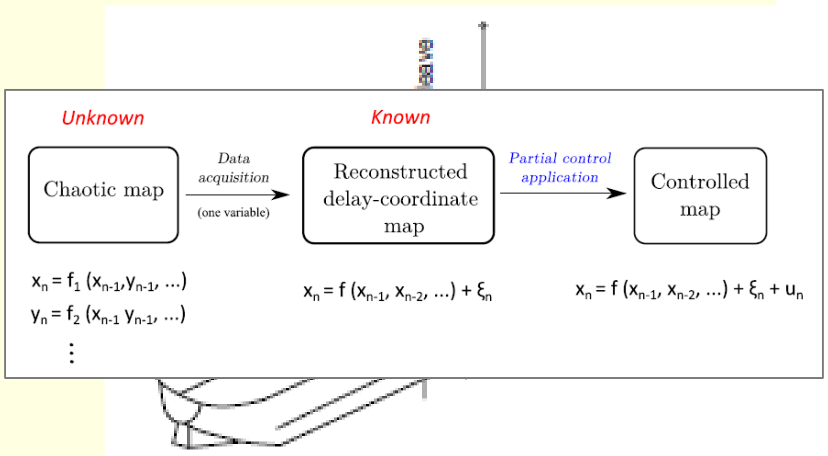




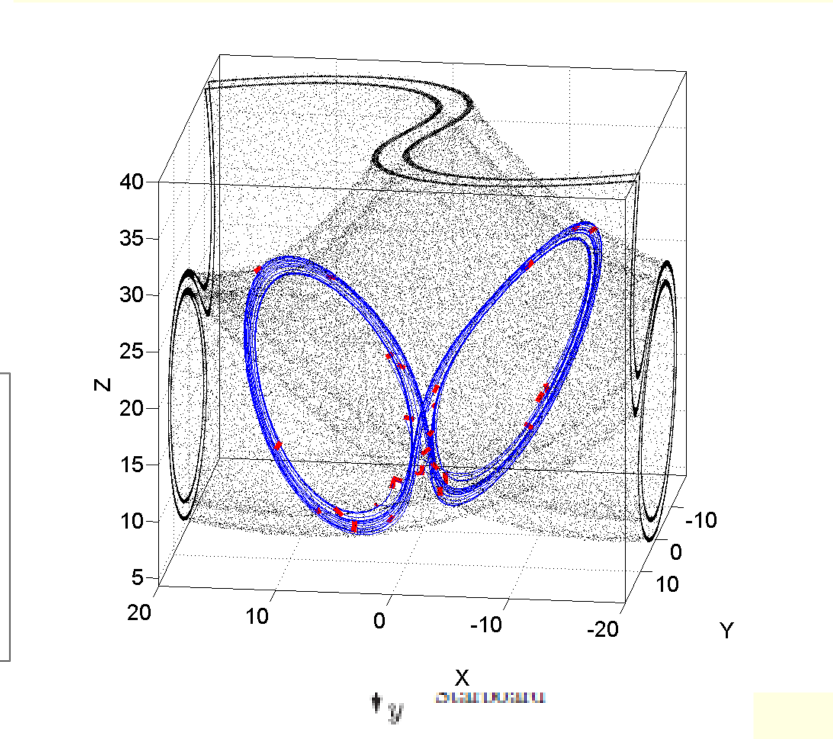
Thank you all for your attention.

Partial control of a ship capsize model

$$\dot{\phi} = \omega$$
$$\dot{\omega} = -\frac{dV}{d\phi} - \bar{b}_1\omega - \bar{b}_2|\omega|\omega + \bar{H}\sin(\bar{\omega}\bar{t})$$



(a)



(b)

Shibabrat Naik and Shane D. Ross. Geometric aproaches in Phase Space Transport and Partial Control of Escaping Dynamics. PhD Thesis. Virginia Tech University.