Universidad
Rey Juan Carlos

Jornada Científica en Homenaje al Prof. Miguel Ángel Fernández Sanjuán por su 60 cumpleaños

Jueves 12 de diciembre de 2019

Salón de Grados del Edificio Departamental II del Campus de Móstoles de la Universidad Rey Juan Carlos

PROGRAMA DE LAS JORNADAS

Sesión de mañana

10:15 Inauguración de las Jornadas

10:30 *"Miguel AF Sanjuán: su vida dedicada a la Ciencia"*
Jesús Miguel Seoane Sepúlveda. Profesor Titular de Física de la URJC.

11:00 *"Grupos de Cayley-Klein: una perspectiva histórica"*
Mariano Santander Navarro. Catedrático de Física de la Universidad de Valladolid

11:30 Pausa de café

12:00 *"Caos y estructuras fractales en sistemas hamiltonianos con escapes"*
Roberto Barrio Gil. Catedrático de Matemática Aplicada de la Universidad de Zaragoza

12:30 *"Miguel Ángel Fernández Sanjuán visto por sus amigos"*
Entre otros, intervendrán:

- Adolfo Azcárraga. Catedrático de Física de la Universidad de Valencia. Presidente de la RSEF
- Alfredo Tiemblo. Profesor de Investigación del CSIC
- Andrés Fernández Díaz. Catedrático de Economía de la UCM
- Juan Luis Vázquez. Catedrático de Matemática Aplicada de la UAM
- Manuel De León. Profesor de Investigación del CSIC
- Arturo Romero. Catedrático de Ingeniería Química de la UCM
- Luis Vázquez Martínez. Catedrático de Matemática Aplicada de la UCM
- José María Pastor. Catedrático de Física y Química de IES
- Samuel Zambrano. Profesor Universitario. Milán. Italia

14:00 Almuerzo en el *"Asador Las Cañas"*
(https://www.restauranteasadorenmostoles.es/es/)

Sesión de tarde

17:00 Intervención de los actuales estudiantes de Doctorado de Miguel Ángel

- Raúl Alelú Paz
- Juan Diego Bernal Fernández
- Roberto Lozano Cardoso
- Alexandre Rodríguez Nieto
- Julia Cantisán Gómez
- Andreu Puy Contreras
- Diego Sánchez Fernández

18:00 Pausa de café

18:30 *"La entropía de Cuencas y la Propiedad de Wada en sistemas dinámicos"*
Alexandre Wagemakers. Profesor Contratado Doctor de la URJC

18:50 *"Caos en osciladores no lineales"*
Mattia T. Coccolo. Profesor Ayudante Doctor de la URJC

19:10 *"El control parcial de sistemas caóticos"*
Rubén Capeáns Rivas. Profesor Visitante de la URJC

19:30 Clausura de las Jornadas
Miguel Ángel Alario. Catedrático de Química Inorgánica de la UCM

21:00 Cena en el Centro de Madrid en el Restaurante *"Los Galayos"*
(www.losgalayos.net)

Jornada Científica en Homenaje al Prof. Miguel Ángel Fernández Sanjuán

Jueves 12 de diciembre de 2019
Salón de Grados del Departamental II de Móstoles

Universidad
Rey Juan Carlos

PHYSICAL REVIEW E, VOLUME 64, 066208

Wada basins and chaotic invariant sets in the Hénon-Heiles system

Jacobo Aguirre, Juan C. Vallejo, and Miguel A. F. Sanjuán
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(Received 25 July 2001; published 27 November 2001)

The Hénon-Heiles Hamiltonian is investigated in the context of chaotic scattering, in the range of energies where escaping from the scattering region is possible. Special attention is paid to the analysis of the different nature of the orbits, and the the invariant sets, such as the stable and unstable manifolds and the chaotic saddle. Furthermore, a discussion on the average decay time associated to the typical chaotic transients, which are present in this problem, is presented. The main goal of this paper is to show, by using various computational methods, that the corresponding exit basins of this open Hamiltonian are not only fractal, but they also verify the more restrictive property of Wada. We argue that this property is verified by typical open Hamiltonian systems with three or more escapes.

DOI: 10.1103/PhysRevE.64.066208 PACS number(s): 05.45.Ac, 05.45.Pq, 95.10.Fh

REVIEWS OF MODERN PHYSICS, VOLUME 81, JANUARY–MARCH 2009

Fractal structures in nonlinear dynamics

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(Published 17 March 2009)

In addition to the striking beauty inherent in their complex nature, fractals have become a fundamental ingredient of nonlinear dynamics and chaos theory since they were defined in the 1970s. Moreover, fractals have been detected in nature and in most fields of science, with even a certain influence in the arts. Fractal structures appear naturally in dynamical systems, in particular associated with the phase space. The analysis of these structures is especially useful for obtaining information about the future behavior of complex systems, since they provide fundamental knowledge about the relation between these systems and uncertainty and indeterminism. Dynamical systems are divided into two main groups: Hamiltonian and dissipative systems. The concepts of the attractor and basin of attraction are related to dissipative systems. In the case of open Hamiltonian systems, there are no attractors, but the analogous concepts of the exit and exit basin exist. Therefore basins formed by initial conditions can be computed in both Hamiltonian and dissipative systems, some of them being smooth and some fractal. This fact has fundamental consequences for predicting the future of the system. The existence of this deterministic unpredictability, usually known as final state sensitivity, is typical of chaotic systems, and makes deterministic systems become, in practice, random processes where only a probabilistic approach is possible. The main types of fractal basin, their nature, and the numerical and experimental techniques used to obtain them from both mathematical models and real phenomena are described here, with special attention to their ubiquity in different fields of physics.

DOI: 10.1103/RevModPhys.81.333 PACS number(s): 05.45.Df, 05.45.Ac, 05.45.Pq

PHYSICAL REVIEW E **89**, 042909 (2014)

Effects of periodic forcing in chaotic scattering

Fernando Blesa,[1,*] Jesús M. Seoane,[2,†] Roberto Barrio,[3] and Miguel A. F. Sanjuán[2]
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[2]*Nonlinear Dynamics, Chaos and Complex Systems Group, Departamento de Física, Universidad Rey Juan Carlos, Tulipán s/n, 28933 Móstoles, Madrid, Spain*
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(Received 6 February 2014; published 17 April 2014)

The effects of a periodic forcing on chaotic scattering are relevant in certain situations of physical interest. We investigate the effects of the forcing amplitude and the external frequency in both the survival probability of the particles in the scattering region and the exit basins associated to phase space. We have found an exponential decay law for the survival probability of the particles in the scattering region. A resonant-like behavior is uncovered where the critical values of the frequencies $\omega \simeq 1$ and $\omega \simeq 2$ permit the particles to escape faster than for other different values. On the other hand, the computation of the exit basins in phase space reveals the existence of Wada basins depending of the frequency values. We provide some heuristic arguments that are in good agreement with the numerical results. Our results are expected to be relevant for physical phenomena such as the effect of companion galaxies, among others.

DOI: 10.1103/PhysRevE.89.042909 PACS number(s): 05.45.Ac, 05.45.Df, 05.45.Pq

International Journal of Bifurcation and Chaos, Vol. 22, No. 6 (2012) 1230010 (9 pages)

DOI: 10.1142/S0218127412300108

TO ESCAPE OR NOT TO ESCAPE, THAT IS THE QUESTION — PERTURBING THE HÉNON–HEILES HAMILTONIAN

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Received December 17, 2010

In this work, we study the Hénon–Heiles Hamiltonian, as a paradigm of open Hamiltonian systems, in the presence of different kinds of perturbations as dissipation, noise and periodic forcing, which are very typical in different physical situations. We focus our work on both the effects of these perturbations on the escaping dynamics and on the basins associated to the phase space and to the physical space. We have also found, in presence of a periodic forcing, an exponential-like decay law for the survival probability of the particles in the scattering region where the frequency of the forcing plays a crucial role. In the bounded regions, the use of the OFLI2 chaos indicator has allowed us to characterize the orbits. We have compared these results with the previous ones obtained for the dissipative and noisy case. Finally, we expect this work to be useful for a better understanding of the escapes in open Hamiltonian systems in the presence of different kinds of perturbations.

Keywords: Nonlinear dynamics and chaos; fractals; numerical simulation of chaotic systems.

ISSN: 0218 – 1274

INTERNATIONAL JOURNAL OF BIFURCATION AND CHAOS IN APPLIED SCIENCES AND ENGINEERING

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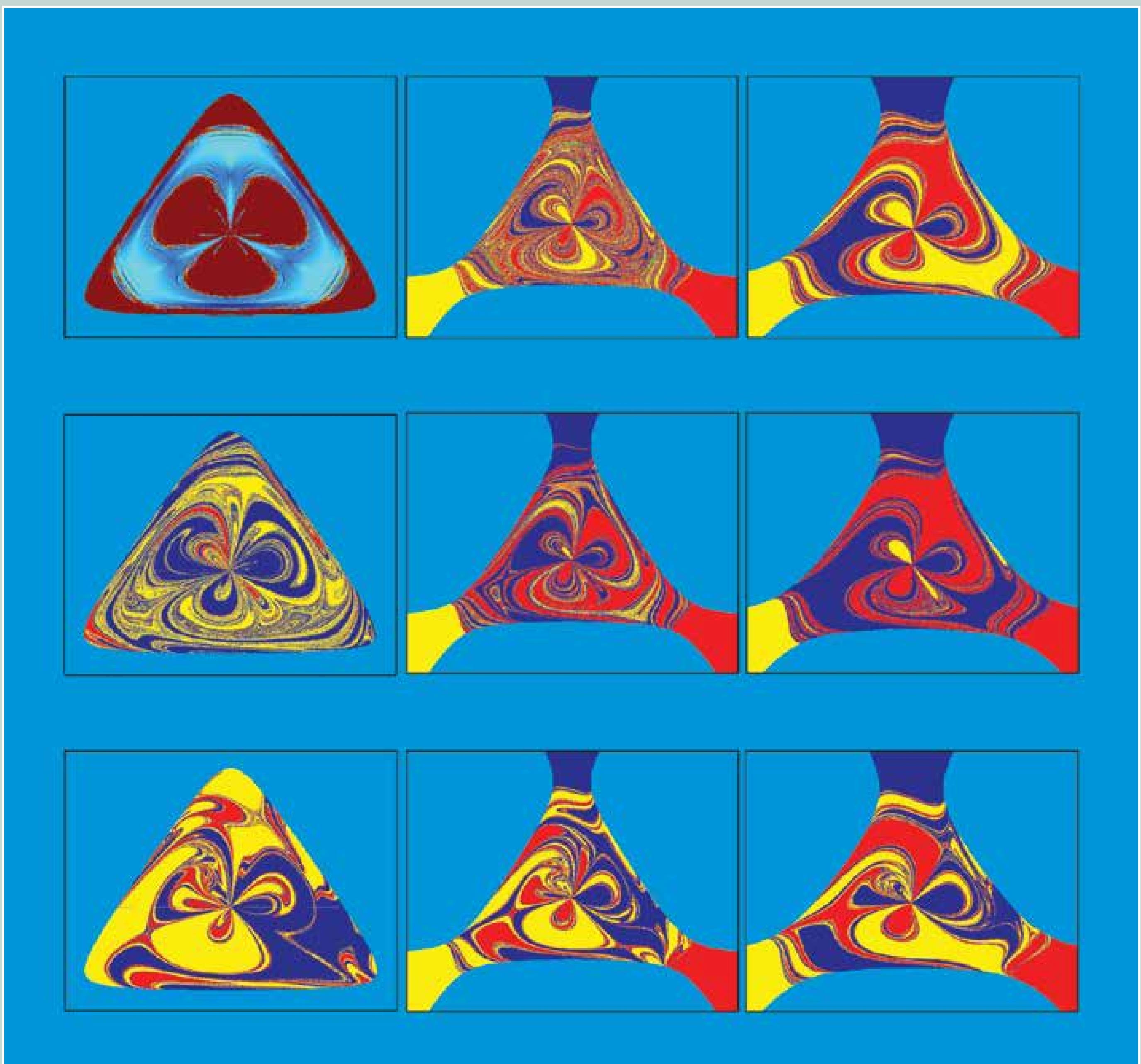


World Scientific

INTERNATIONAL JOURNAL OF BIFURCATION AND CHAOS

VOL. 22, NO. 6 (JUNE 2012)


SALIDA

Caos y estructuras fractales en sistemas hamiltonianos con escapes

Roberto Barrio

GME – University of Zaragoza, SPAIN

Homenaje al Prof. Miguel Ángel Fernández Sanjuán
Móstoles, 12 Diciembre 2019

Bifurcaciones y Caos en sistemas Hamiltonianos

Roberto Barrio, Fernando Blesa, Sergio Serrano

GME – University of Zaragoza, SPAIN

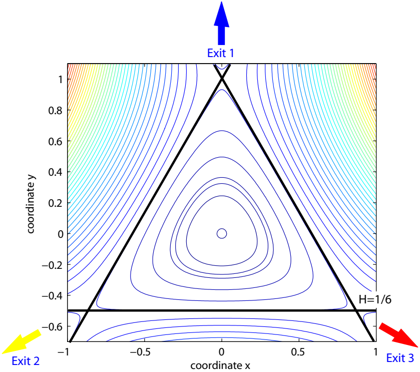
Complejidad 2008
(Complejidad'08) Móstoles, 20–21 Noviembre 2008

The Hénon-Heiles Hamiltonian (1964)

$$\mathcal{H} = \frac{1}{2}(X^2 + Y^2) + \frac{1}{2}(x^2 + y^2) + \left(x^2 y - \frac{1}{3}y^3\right)$$

Symmetries:

- the spatial group is a dihedral group D_3
- the complete symmetry group is $D_3 \times \mathcal{T}$ ($\mathcal{T}$ is a $\mathbb{Z}_2$ symmetry, *the time reversal symmetry*)



[1] Hénon, M.; Heiles, C. (1964). “The applicability of the third integral of motion: Some numerical experiments”. The Astronomical Journal. 69:73-r79

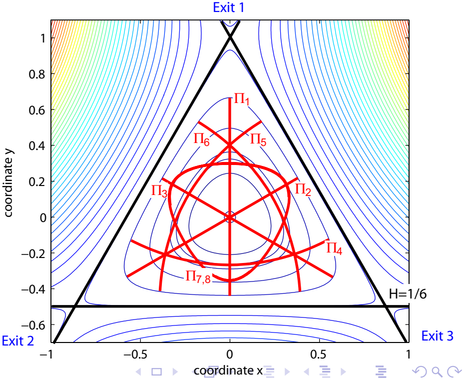
The Hénon-Heiles Hamiltonian

Theorem (Weinstein (1973))

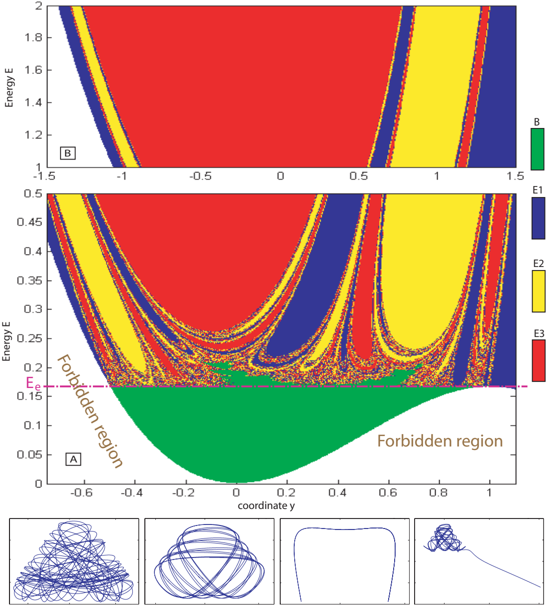
If the Hamiltonian $\mathcal{H}(\mathbf{x}, \mathbf{X})$ *is of class* $\mathcal{C}^2$ *near* $(\mathbf{x}, \mathbf{X}) = (0, 0)$*, where* $\mathbf{x}, \mathbf{X} \in \mathbb{R}^n$*, and the Hessian matrix* $\mathcal{H}_{**}(0, 0)$ *is positive definite, then for* ε *sufficiently small any energy surface* $\mathcal{H}(\mathbf{x}, \mathbf{X}) = \mathcal{H}(0, 0) + \varepsilon^2$ *contains at least n periodic orbits of the corresponding Hamiltonian equations whose periods are close to those of the linear system* $\dot{\mathbf{z}} = J\mathcal{H}_{**}(0, 0)\mathbf{z}$*.*

Nonlinear normal modes:

- from Weinstein's theorem at least 2
- from the symmetries 8: Π_i, $i = 1, \ldots, 8$ (Churchill *et al.* (1979))



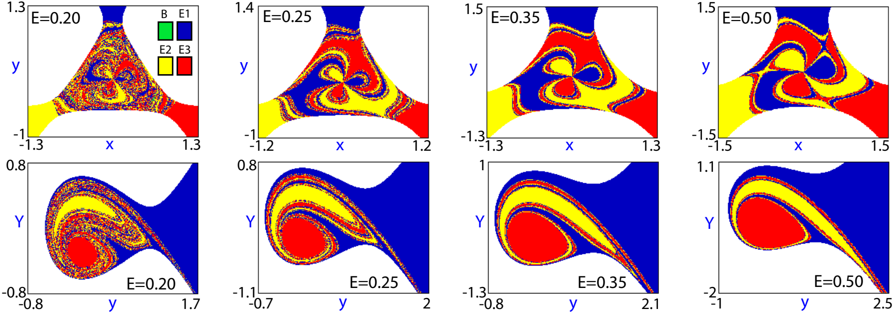
Escape basins: plane (y, E)



- for $\mathcal{H} < 1/6$ all orbits are bounded.
- for $1/6 < \mathcal{H} \lesssim 0.22$ most orbits are escape orbits and some KAM tori persist.
- for $0.22 \lesssim \mathcal{H}$ no KAM tori and all orbits are escape orbits **(?)**.

Escape basins

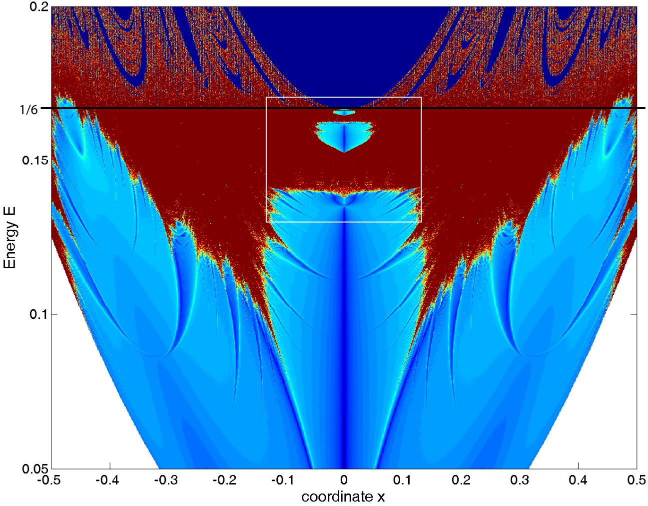
- For any value of E we have fractal exit basins.
- The fractality decreases with E.



- Wada basins: The basins have the Wada property[2]

[2] J. Aguirre, J. C. Vallejo, and M. A. F. Sanjuán, Phys. Rev. E 64, 066208 (2001)

Fractal structures near the critical energy level: Π_1



Below escape energy:

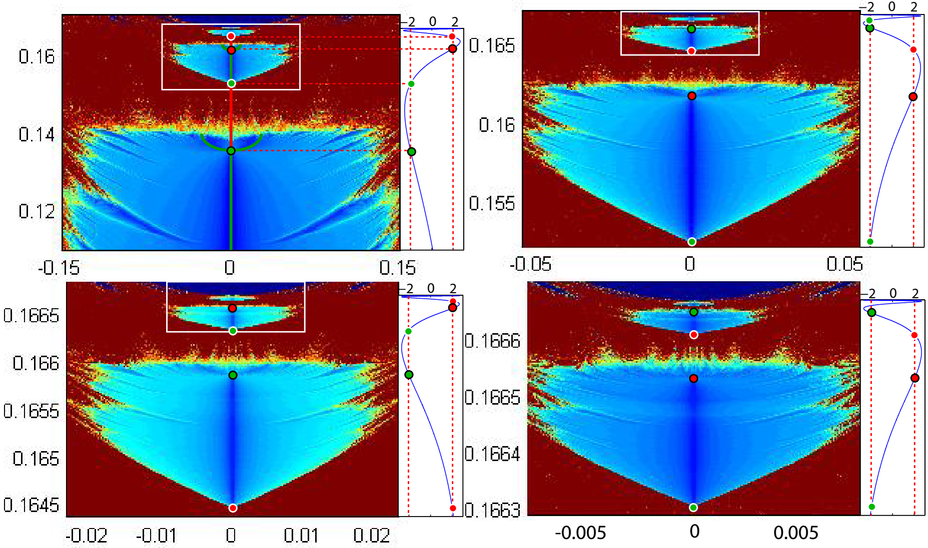
- blue regular
- red chaos.

Above escape energy:

- dark blue escape orbits.
- red escape with transient chaos.

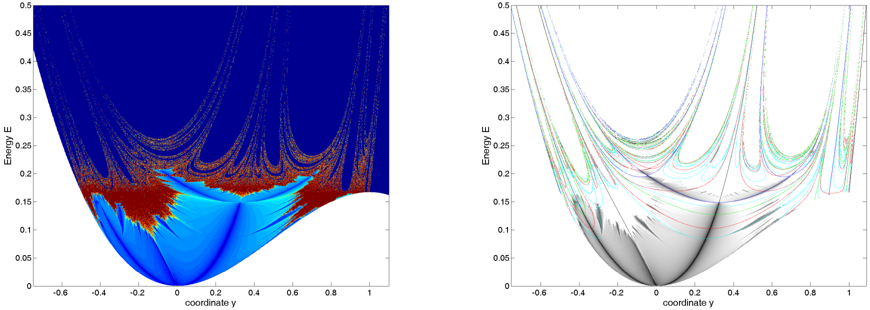
- Π_1 stability varies as E approaches the critical value.

Fractal structures near the critical energy level



Fractal bounded structures and symmetric p.o.

- ♢ KAM tori disappear on y-axis around $E \approx 0.2113$.
- ♣ Periodic orbits.
- ♣ OFLI2 chaos indicator.

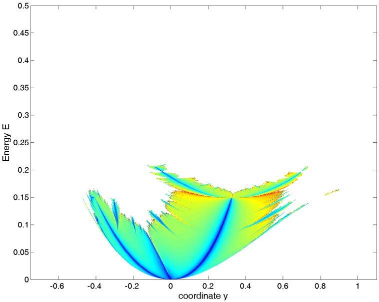


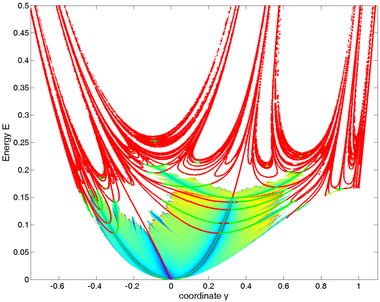
R. Barrio, F. Blesa, S. Serrano. EPL **82** (1) 10003 (2008).

Fractal bounded structures and symmetric p.o.

- Fat-fractal exponent of the regular region: $\gamma = 0.637(\pm 0.056)$.

- Red: unstable p.o.
- Green: stable p.o.
- Small zones of stable periodic orbits.

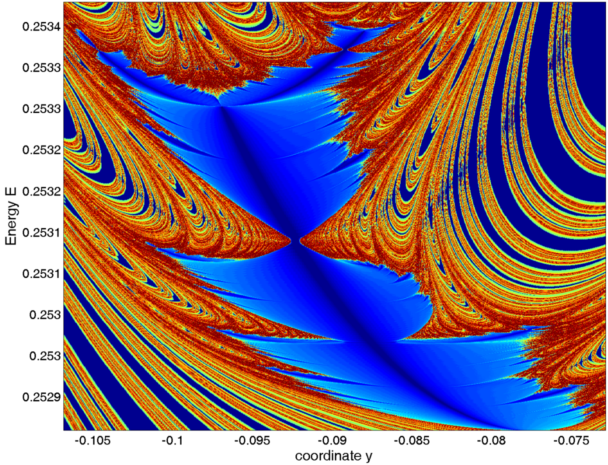




R. Barrio, F. Blesa, S. Serrano. EPL **82** (1) 10003 (2008).

Fractal and regular bounded structures

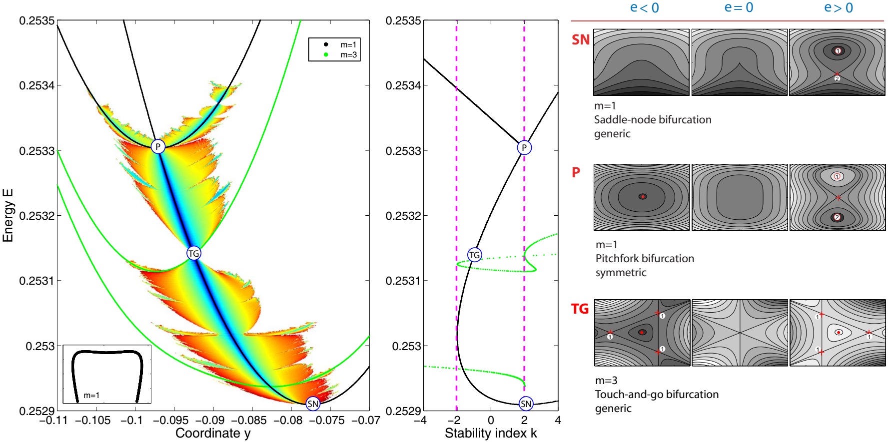
In the escape region



Above the escape energy:

- Safe regions: bounded structures in the escape region.
- Small regular region around $E \approx 0.253$.
- Self-similar regions with chains of bifurcations inside.

Bifurcations: safe region



R. Barrio, F. Blesa, S. Serrano, New Journal of Physics, **11** (5) 053004 (2009).

Adding perturbations: to escape or not to escape ...[3]

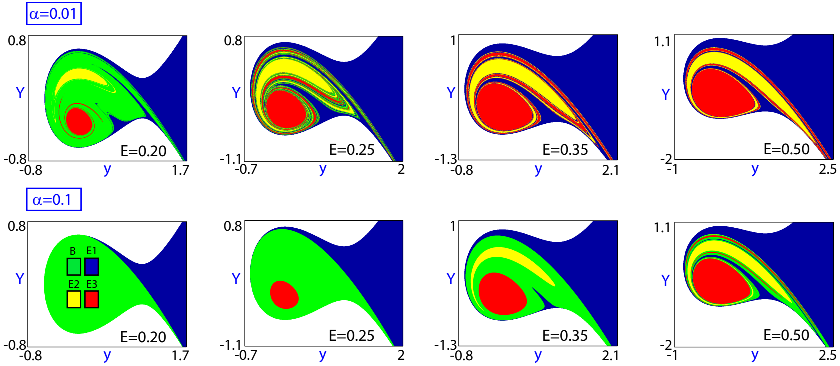
- Unperturbed
- Dissipation
- Periodic driving
- White Gaussian noise

$$\left.\begin{aligned}\ddot{x} &= -\frac{\partial \mathcal{H}_0}{\partial x} - \alpha_x \dot{x} + A_x \sin(\omega_x t) + \sqrt{2\varepsilon}\xi(t)\\ \ddot{y} &= -\frac{\partial \mathcal{H}_0}{\partial y} - \alpha_y \dot{y} + A_y \sin(\omega_y t) + \sqrt{2\varepsilon}\eta(t)\end{aligned}\right\}$$

[3] "To escape or not to escape, that is the question–Perturbing the Henon-Heiles Hamiltonian", F. Blesa, J. Seoane, R. Barrio, M.A. Sanjuán, IJBC, Vol. 22, No. 6 (2012).
"Effects of periodic forcing in chaotic scattering", F. Blesa, J. Seoane, R. Barrio, M.A. Sanjuán, PRE 89, 042909 (2014).

Dissipation

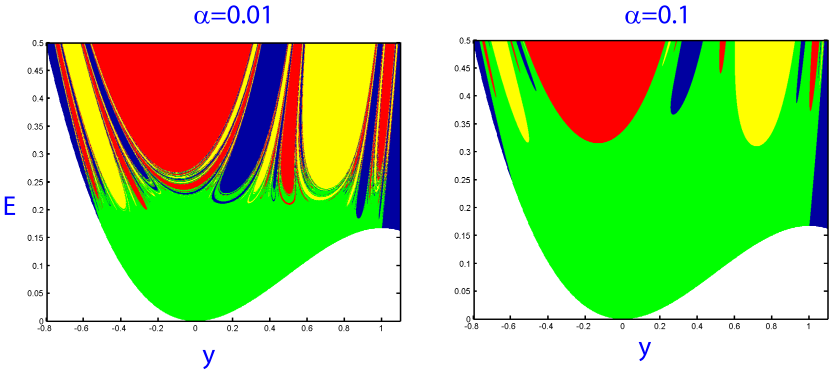
- Above $\alpha = 0.01$ and below $\alpha = 0.1$



- As the dissipation grows, the Wada property appears later.
- There are more orbits that don't escape.

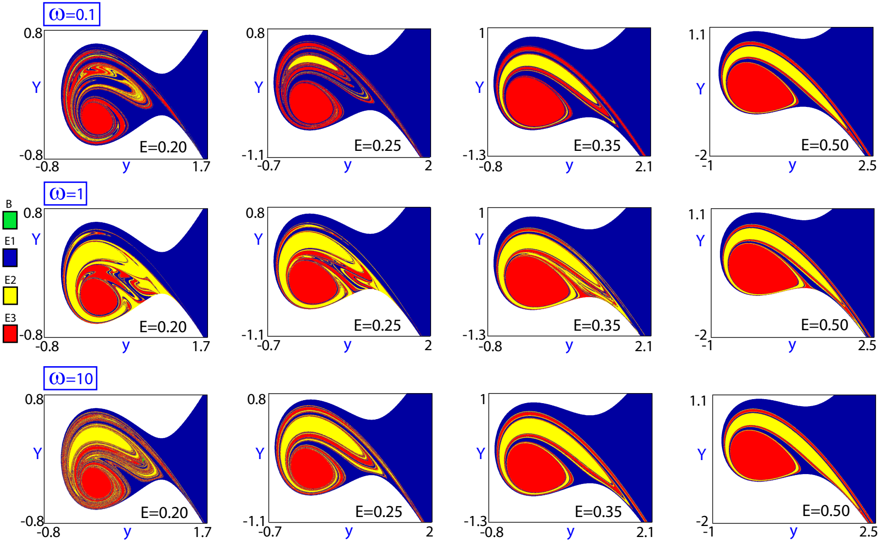
Dissipation: changing the initial energy

- Left: $\alpha = 0.01$ and right: $\alpha = 0.1$

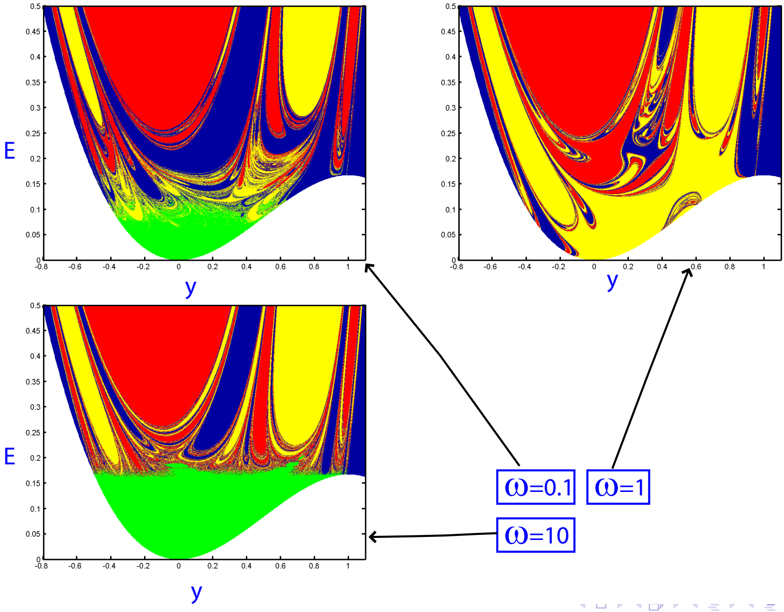


- The basins are not mixed when the dissipation grows.

Periodic driving: $A = 0.1$

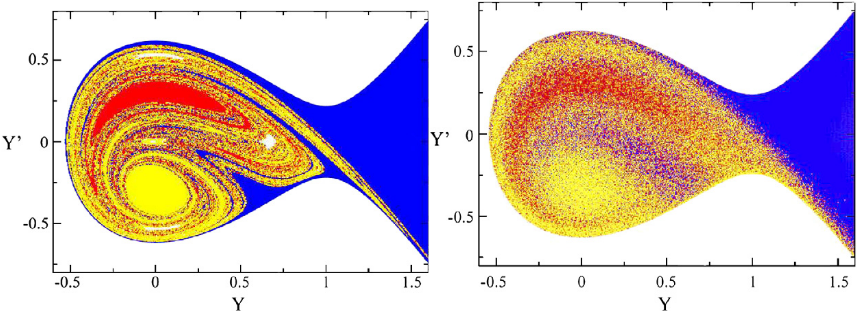


Periodic driving: $A_x = A_y = 0.1$



Noise: $E = 0.19$ and $\varepsilon = 0.002$

Noiseless case. $\epsilon = 0.002$



The basins appear smeared because of the noise effect.

J. M. Seoane, L. Huang, M. A. F. Sanjuán, and Y. C. Lai, Phys. Rev. E 79, 047202 (2009).

Adding perturbations: to escape or not to escape ...

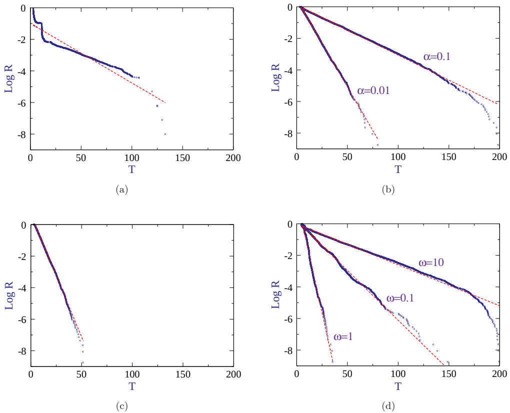


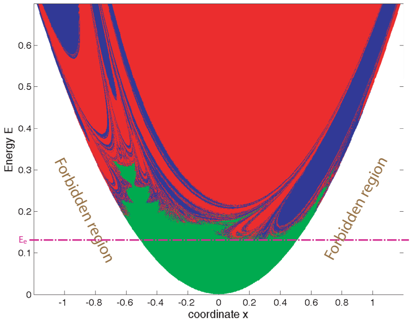
Fig. 4. Typical exponential decay law for the particles remaining in the scattering region. R denotes the fraction of particles remaining in the scattering region. Here, we shoot 5×10^3 with energy $E = 0.2$ from $(x_0, y_0) = (0, -0.5)$ and $\theta \in (0, 2\pi)$. (a) Algebraic law of the unperturbed system. (b) In presence of dissipation. The dissipative parameter is $\alpha = 0.01$ or $\alpha = 0.1$. (c) Due to the noise effects. The intensity of the noise $\varepsilon = 0.01$ and (d) with a periodic driving. The forcing amplitude is $A = 0.1$ and the forcing frequency is $\omega = 1$ (resonant case), $\omega = 0.1$ or (c) $\omega = 10$. The oscillations around the straight line obtained from the linear regression of the numerical data is due to the value of the chosen frequency ω.

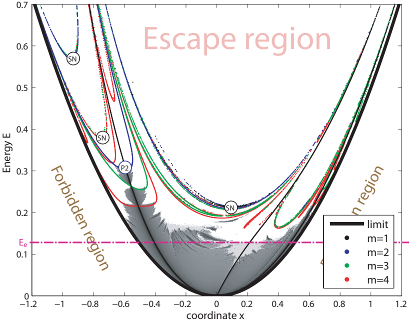
- And soon ... relativistic effects (Sanjuán, Bernal, Seoane, Blesa, Barrio)

Other open Hamiltonians: the Barbanis potential

$$\mathcal{H} = \frac{1}{2}(X^2 + Y^2) + \frac{1}{2}(x^2 + y^2) - xy^2.$$

- Two exits
- Applications in quantum dynamics and to model $S_1 \leftarrow S_0$ fluorescence excitation of benzophenone.





But I am mathematician $\rightarrow$
$\rightarrow$ Computer Assisted Proofs: interval arithmetic

Definition: Interval Newton Operator

Let $y_0 \in [y]$ (*interval*). Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $\mathcal{C}^{(1}$, such that $f' \neq 0$ in $[y]$.

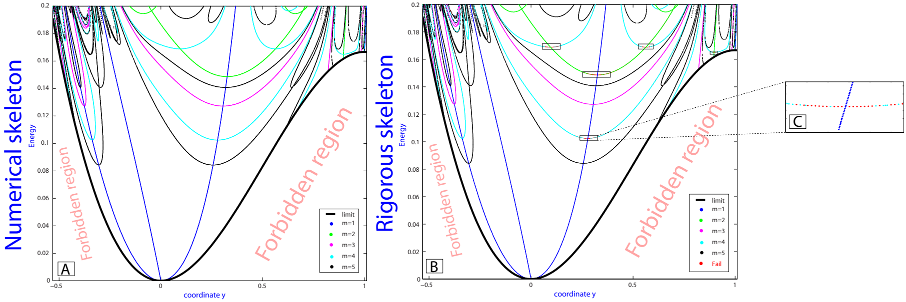
$$N(y_0, [y], f) = y_0 - \frac{f(y_0)}{f'([y])}$$

Theorem

- If $y_1, y_2 \in [y]$, and $f(y_1) = f(y_2)$, then $y_1 = y_2$.
- If $N(y_0, [y], f) \subset [y]$, then $\exists! y^* \in [y]$ such that $f(y^*) = 0$.
- If $N(y_0, [y], f) \cap [y] = \emptyset$, then $f(y) \neq 0$ in $[y]$.
- If $y_1 \in [y]$ and $f(y_1) = 0$, then $y_1 \in N(y_0, [y], f)$.

We have to transform our problem into a zero-finding problem.
Use of the CAPD software (Krakow)

Graphical theorem[4].

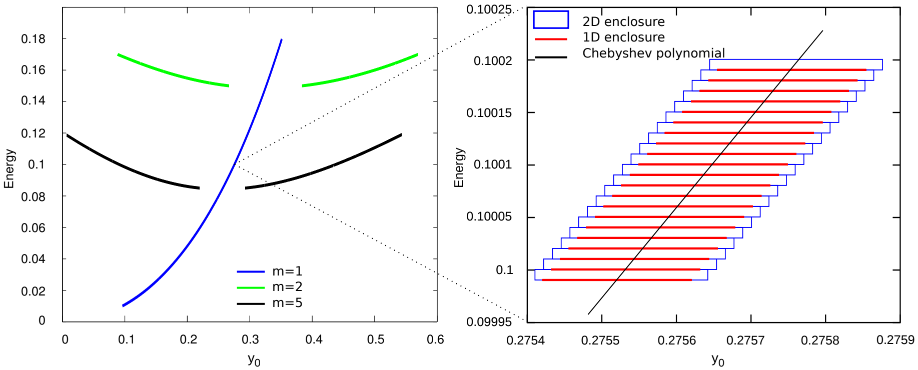


Theorem ≈ 25000 proofs

Each point (except red points) represents the rigorous initial conditions of a unique periodic orbit in an interval of radius 10^{-8}, whose multiplicity will be in $\{1, 2, 3, 4, 5\}$, according to the color (except red points).

[4] R Barrio, M Rodriguez, F Blesa, "Computer-assisted proof of skeletons of periodic orbits", *Computer Physics Communications* 183 (2012), 80-85

Graphical theorem[5]



Theorem

Inside each colored area there exist a continuous family of periodic orbits of multiplicity 1, 2 or 5 according to the color.

[5] Systematic Computer Assisted Proofs of periodic orbits of Hamiltonian systems R Barrio, M Rodríguez Communications in Nonlinear Science and Numerical Simulation 19 (8), 2660-2675, 2014.

Analytical theorem

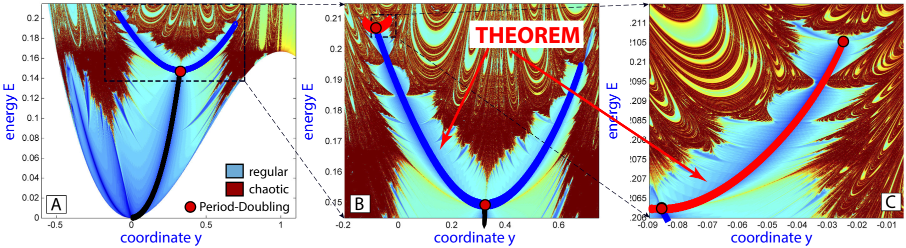
Theorem. Family $m = 1$

For the Henon-Héiles system, let the energy E be in the interval $[0.01, 0.18]$. We consider $\widetilde{E}$ the linear transformation of E into the interval $[-1, 1]$. We define $p^*(\widetilde{E}) = \sum_{i=0}^{10} c_i T_i(\widetilde{E})$, where T_i are the Chebyshev polynomials. Then for each E in $[0.01, 0.18]$ there exists a unique periodic orbit of multiplicity 1, whose initial conditions are:

$$y_0 = p(\widetilde{E}) \pm \varepsilon$$
$$x_0 = Y_0 = 0$$
$$X_0 = X(x_0, y_0, Y_0, E) \text{ (according to Hamiltonian equation)}$$

where $0 \leq \varepsilon \leq 10^{-4}$.

Analytical theorem of the existence of the KAM tori[6]



- Computer assisted proof of the existence of the families of periodic orbits.
- Computer assisted proof of the existence of multiplicity 1, 2, 3 and 4 bifurcations of periodic orbits.
- Computer assisted proof of the existence of the invariant tori.

[6] "Systematic Computer-Assisted Proof of branches of stable elliptic periodic orbits and surrounding invariant tori", D Wilczak, R Barrio, SIAM Journal on Applied Dynamical Systems 16 (3), 1618-1649, 2017.

¡¡Felicidades !!
Miguel Ángel