Universidad Rey Juan Carlos

Móstoles, Madrid, España

Osciladores no lineales

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Jornada Científica en Homenaje al
Prof. Miguel Ángel Fernández Sanjuán
por su 60 cumpleaños
12 diciembre 2019

¿Porqué osciladores lineales?

TESIS DOCTORAL

CONTRIBUCION AL ESTUDIO ANALITICO DE ALGUNAS PROPIEDADES DE OSCILADORES NO LINEALES

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Licenciado en Ciencias Físicas
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Universidad de Valladolid

Presentada en la

Facultad de Ciencias
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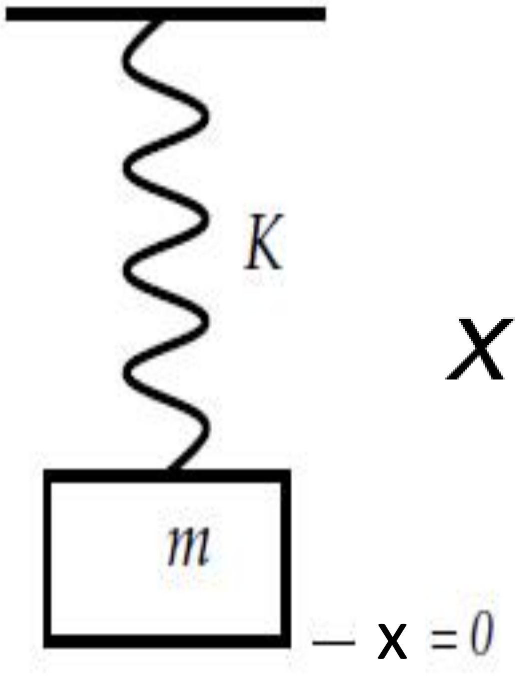
para la obtención del

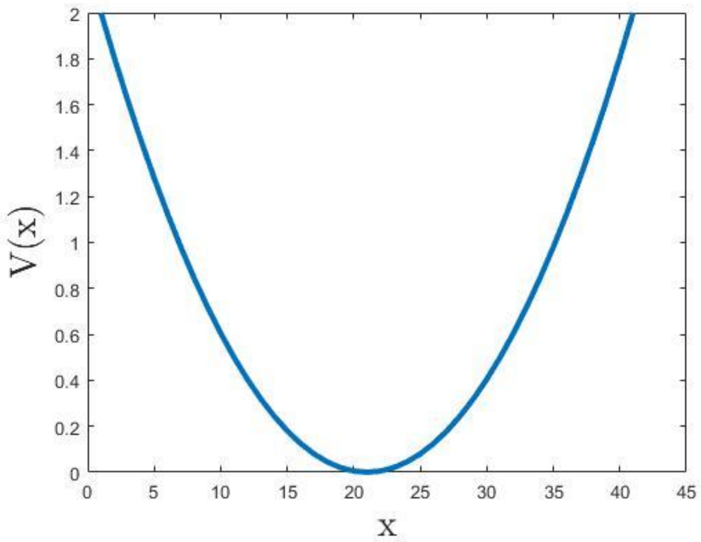
Grado de Doctor en Ciencias Físicas

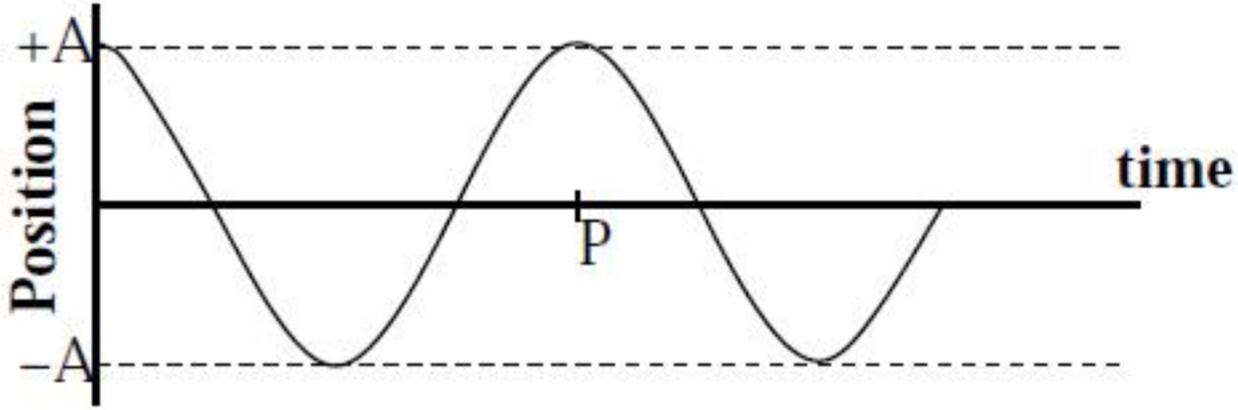
Madrid, Septiembre de 1990

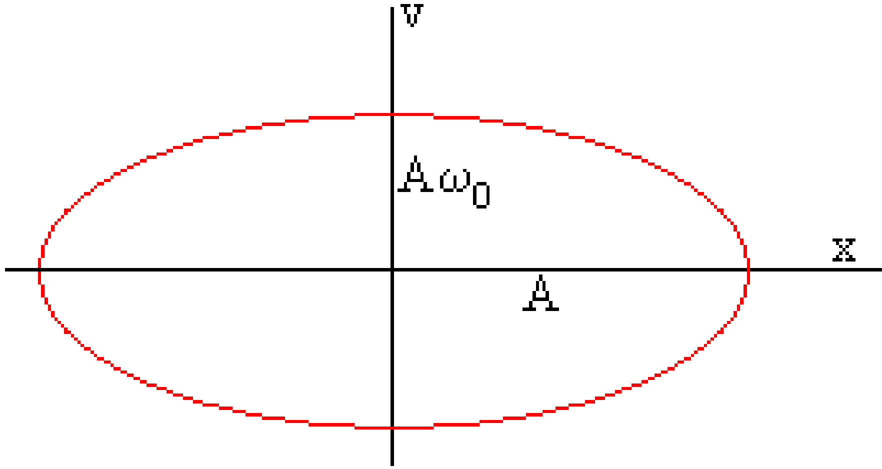


$$m\ddot{x} = -kx$$

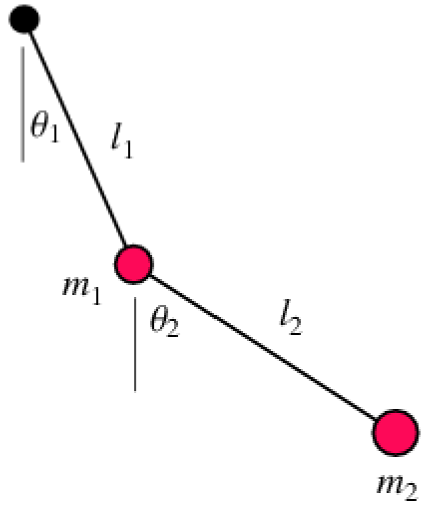
$$x = A \sin(\omega t + \phi)$$



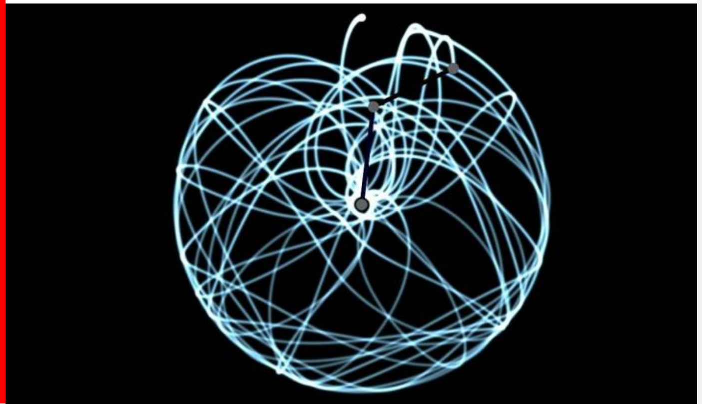


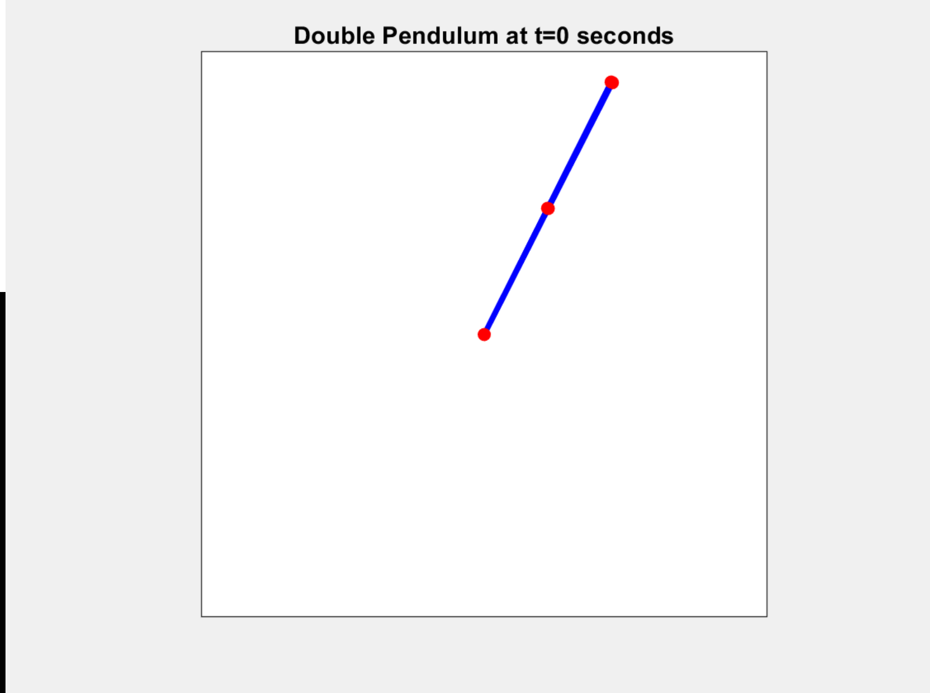


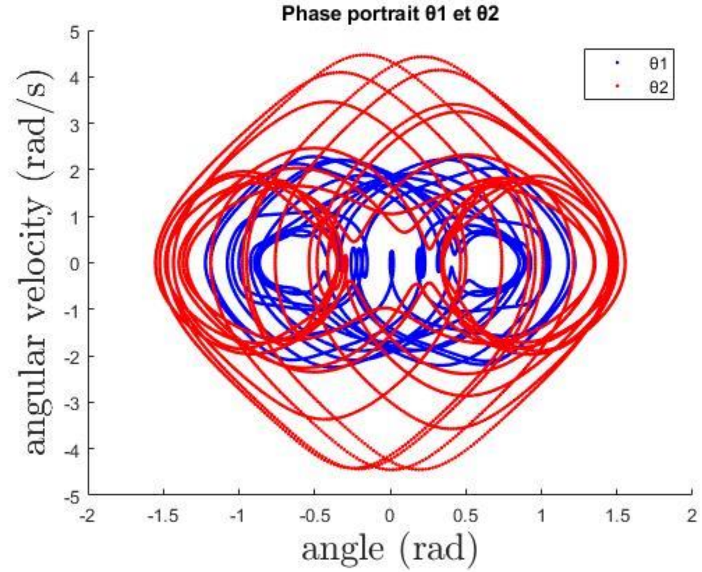
Osciladores no lineales: el péndulo doble



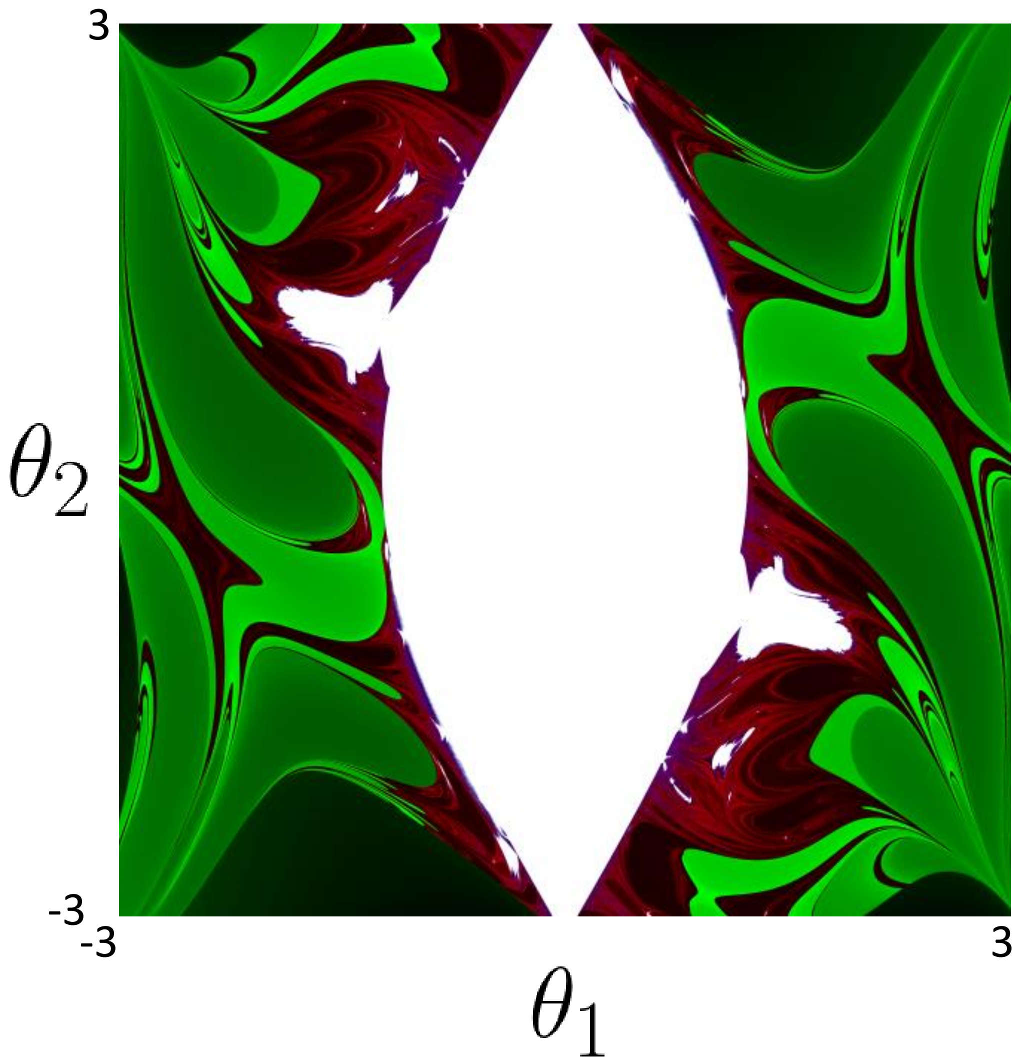
$$x_1 = l_1 \sin\theta_1$$
$$x_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2$$
$$y_1 = -l_1 \cos\theta_1$$
$$y_2 = -l_1 \cos\theta_1 - l_2 \cos\theta_2$$







El péndulo doble: ver la complejidad

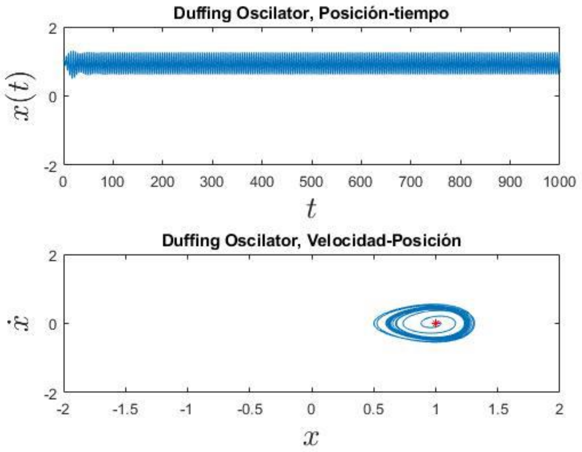


Tiempos antes de dar la primera vuelta:

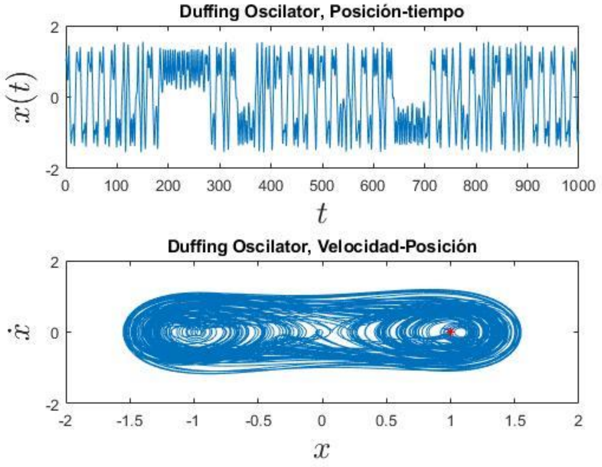
- $10\sqrt{(l/g)}$ (verde)
- $100\sqrt{(l/g)}$ (rojo)
- $1000\sqrt{(l/g)}$ (morado)
- $10000\sqrt{(l/g)}$ (azul).
- Condiciones iniciales que no llevan a dar una vuelta antes de $10000\sqrt{l/g}$ se han dejado blancas

Osciladores no lineales: ecuación de Duffing

_γ=0.1, F=0.1, ω=1.4_

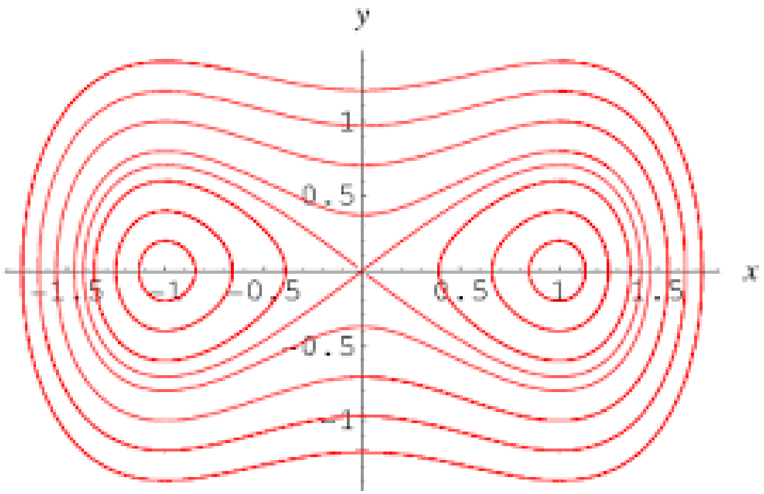


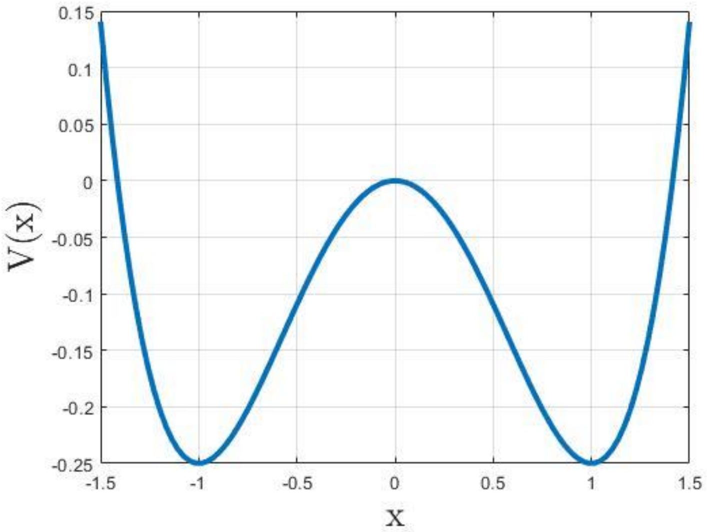
_γ=0.1, F=0.35, ω=1.4_



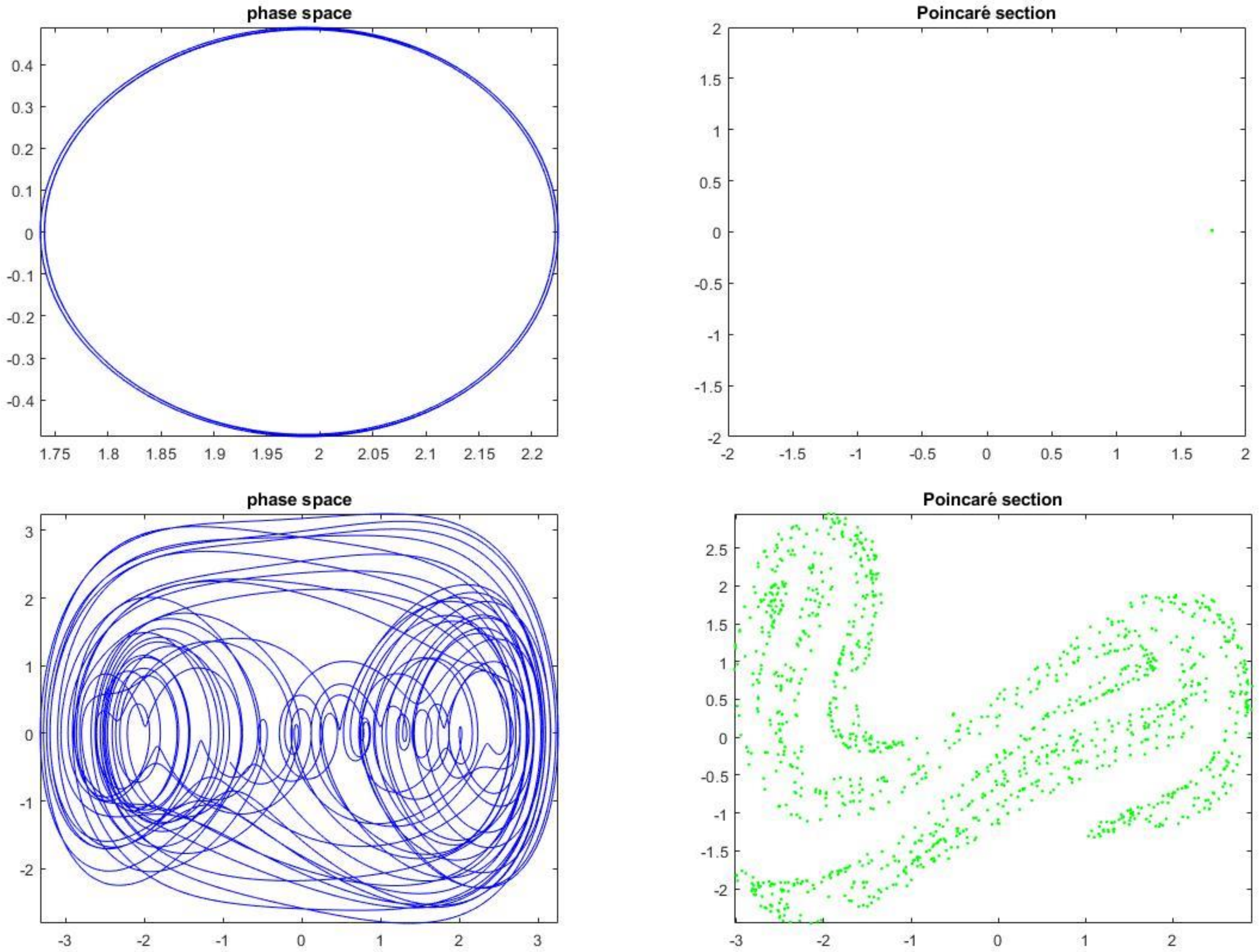
$$\ddot{x} - x + x^3 + \gamma v = F \cos \omega t$$

$$V(x) = -\frac{x^2}{2} + \frac{x^4}{4}$$





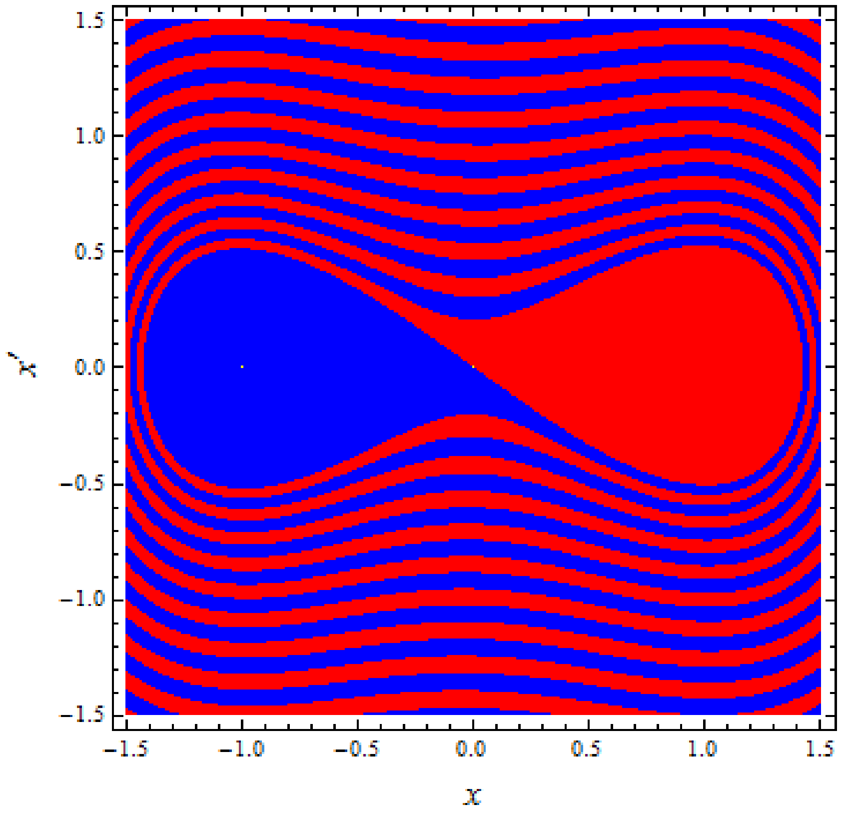
Ecuación de Duffing: sección de Poincaré

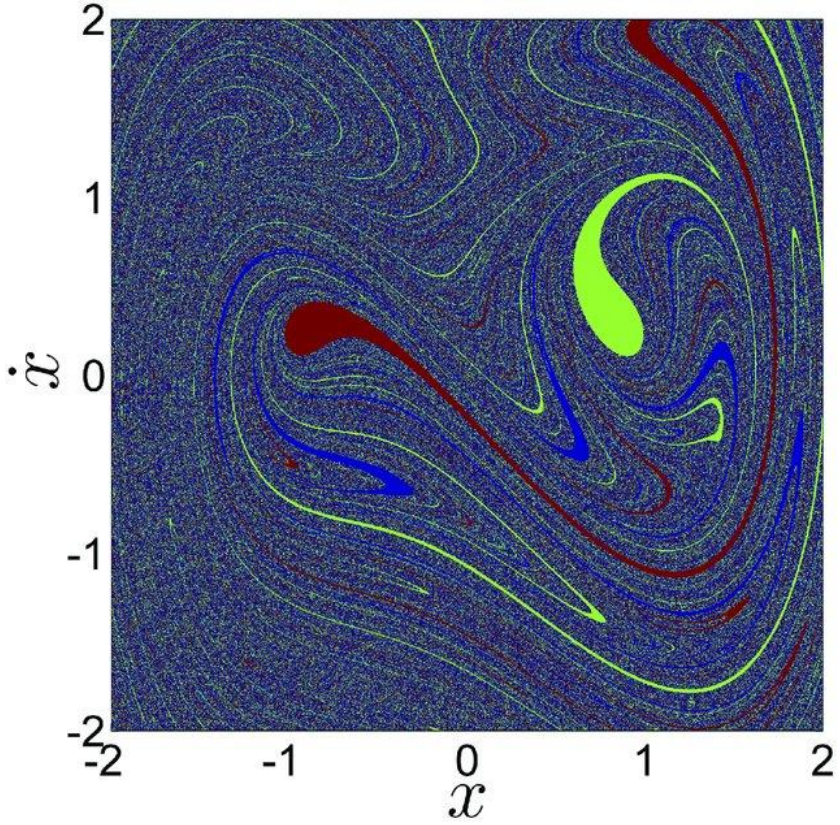


Periodicidad

Caos

Ecuación de Duffing: Cuencas de atracción





Control de fase en modelo de masa-muelle


International Journal of Bifurcation and Chaos, Vol. 23, No. 12 (2013) 1330042 (13 pages)

DOI: 10.1142/S0218127413300425

PHASE CONTROL IN THE MASS-SPRING MODEL WITH NONSMOOTH STIFFNESS AND EXTERNAL EXCITATION

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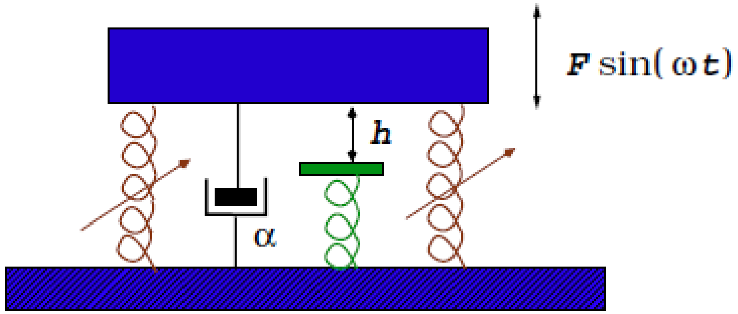
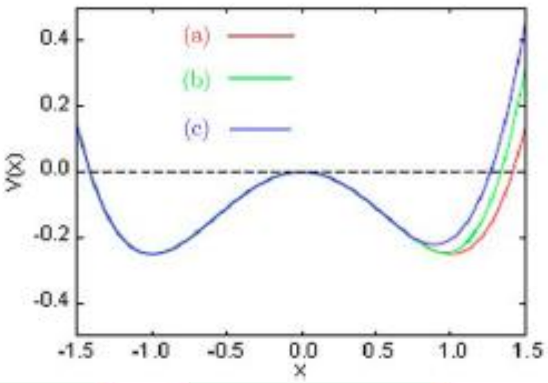
The control of chaotic dynamics in a nonlinear mass-spring model with nonsmooth stiffness is analyzed here. This is carried out by applying the phase control technique, which uses a periodic perturbation of a suitable phase ϕ. For this purpose, we take as prototype model a system consisting of a double-well potential with an additional spring component, which acts into the system only for large enough displacements. The crucial role of the phase is evidenced by using numerical simulations and also by using analytical methods, such as the Melnikov analysis. We expect that our results might be fruitful with implications in some mechanical problems such as suspension of vehicles, among others, where similar models are extensively used.

Keywords: Phase control; nonlinear oscillations; Melnikov criterion; chaos; nonsmooth dynamical systems.

$$\ddot{x} + \alpha\dot{x} - x + x^3 + k(x-h)\Theta(x-h)$$
$$+ \varepsilon\sin(r\omega t + \phi) = F\sin(\omega t).$$





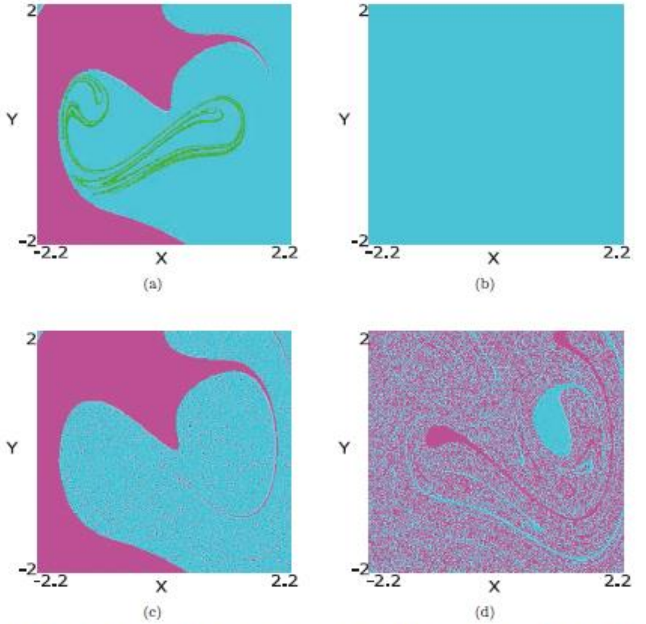


Fig. 10. Plots of the basins of attraction of our system (in the phase plane $(x, y = \dot{x})$) for (a) $F = 0.3$, $k = 0.45$ and $h = 0.1$ without control in which chaotic motions take place, (b) $F = 0.26$, $k = 0.45$ and $h = 0.1$ in which the regular motions take place, (c) and (d) with control ($\varepsilon = 0.2$): (c) $\phi = \pi$ and (d) $\phi = 0$. Finally, we observe the important influence of the phase since the basin structure is altered by the phase effects.

El fenómeno de la resonancia, aunque bien conocido, es todavía bajo estudio

RESONANCIAS NO LINEALES

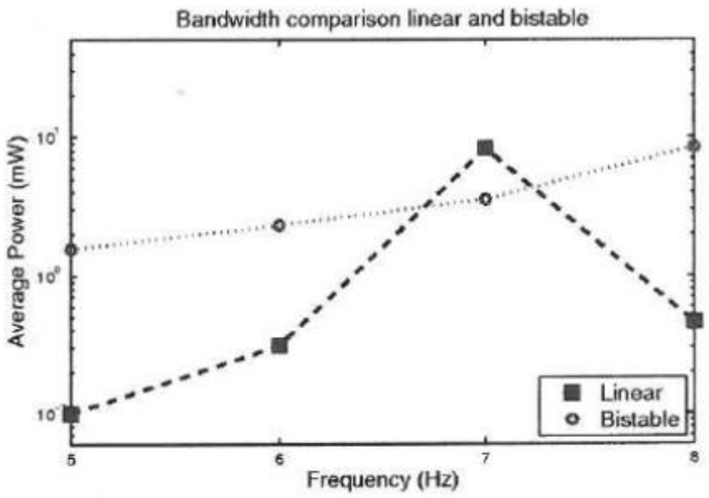
• Stochastic resonance	→	• Ruido blanco y forzamiento periódico
• Chaotic resonance	→	• Signal caótica y forzamiento periódico
• Coherence resonance	→	• Ruido sin forzamiento externo
• Vibrational resonance	→	• Dos forzamiento armónicos $\Omega > \omega$

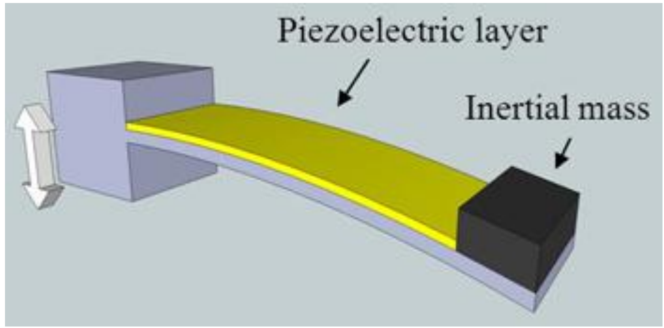
Rajasekar, S., Sanjuán, M.A.F.: Nonlinear Resonances. Springer, Cham (2016)

Usar la energía ambiental para alimentar aparados eléctricos portátiles

La posibilidad de no depender estrictamente solo de las baterías

Los aparados funcionen por más tiempo de forma más sostenible.





Energy harvesting: resonancia vibratoria

International Journal of Bifurcation and Chaos, Vol. 24, No. 6 (2014) 1430019 (7 pages)

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Energy Harvesting Enhancement by Vibrational Resonance

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The idea to use environmental energy to power electronic portable devices is becoming popular in recent years. In fact, the possibility of not relying only on batteries can provide dev longer operating periods in a fully sustainable way. Vibrational kinetic energy is a reliable and widespread environmental energy, that makes it a suitable energy source to exploit. In this paper, we study the electrical response of a bistable system, by using a double-well Duffing oscillator, connected to a circuit through piezoceramic elements and driven by both a low (LF) and a high frequency (HF) forcing, where the HF forcing is the environmental vibration, while the LF is controlled by us. The response amplitude at low-frequency increases, reaches a maximum and then decreases for a certain range of HF forcing. This phenomenon is called vibrational resonance. Finally, we demonstrate that by enhancing the oscillations we can harvest more electric energy. It is important to take into account that by doing so with a forcing induced by us, the amplification effect is highly controllable and easily reproducible.

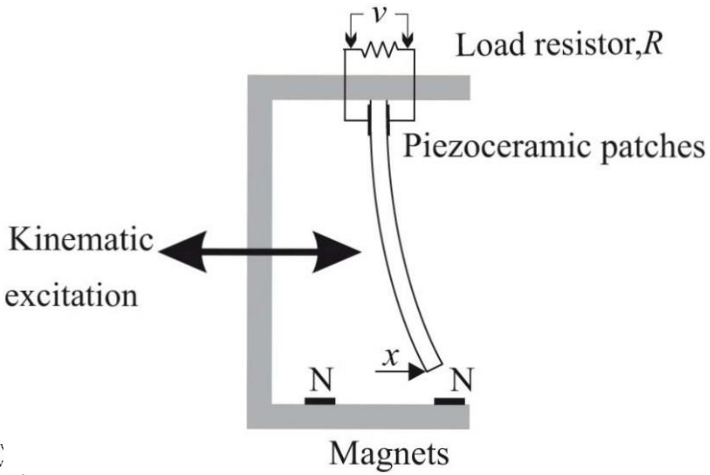
Keywords: Energy harvesting; vibrational resonance.

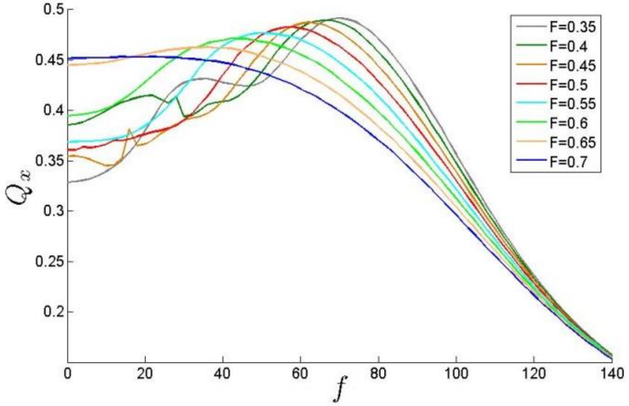
1. Introduction

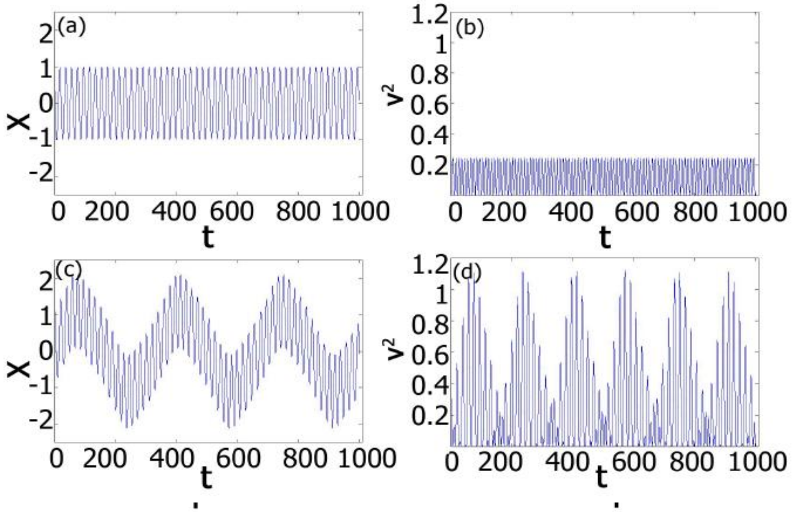
In the last few years, a quick development has occurred in the miniaturization capability of electronic devices. On the other hand, the same improvement speed has not been comparable for the energy density available in batteries that provide the power for such devices, when operating in stand-alone configurations [Paradiso & Starner, 2005]. Thus, the possibility to overcome the limitations related to the power requirement of small electronic components has become an important research field. One recent idea is to power such small electronic devices by using energy available in their environment. This is the core of so-called *energy harvesting*. The main goal that energy harvesting aims to achieve is to reduce the requirement of an external source as well as the maintenance costs for periodic battery replacement and the chemical waste of conventional batteries. Due to its diffusion, an interesting possibility has received growing attention, i.e. converting the micro-kinetic energy, mostly available as random motion often

$$\ddot{x}+\mu\dot{x}-\alpha x(1-x^2)-\chi v = F\cos\omega t+f\cos\Omega t$$

$$\dot{v}+\lambda v-\kappa\dot{x}=0 \qquad \Omega >> \omega$$







Energy harvesting: optimización

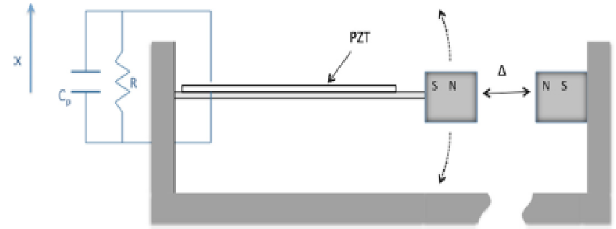
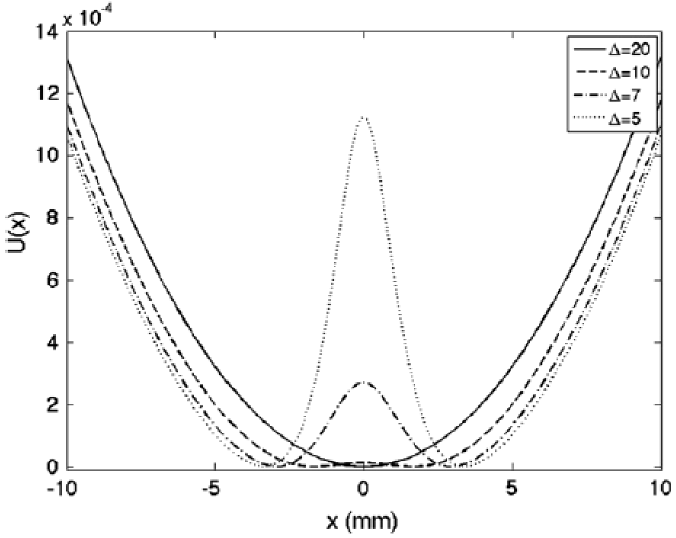


FIG. 1: This figure plots the scheme of the harvester considered.

$$m\ddot{x} + \frac{dU(x)}{dx} + \gamma\dot{x} + K_v v(t) = F\cos(\omega t) + f\cos(\Omega t),$$

$$\dot{v} + v(t)/\tau_p - \kappa_c\dot{x} = 0, \qquad \Omega >> \omega$$

$$U(x) = \frac{1}{2}K_{eff}x^2 + (ax^2 + b\Delta^2)^{-3/2},$$

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Optimizing the Electrical Power in an Energy Harvesting System

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In this paper, we study the vibrational resonance (VR) phenomenon as a useful mechanism for energy harvesting purposes. A system, driven by a low frequency and a high frequency forcing, can give birth to the vibrational resonance phenomenon, when the two forcing amplitudes resonate and a maximum in amplitude is reached. We apply this idea to a bistable oscillator that can convert environmental kinetic energy into electrical energy, that is, an *energy harvester*. Normally, the VR phenomenon is studied in terms of the forcing amplitudes or of the frequencies, that are not always easy to adjust and change. Here, we study the VR generated by tuning another parameter that is possible to manipulate when the forcing values depend on the environmental conditions. We have investigated the dependence of the maximum response due to the VR for small and large variations in the forcing amplitudes and frequencies. Besides, we have plotted color coded figures in the space of the two forcing amplitudes, in which it is possible to appreciate different patterns in the electrical power generated by the system. These patterns provide useful information on the forcing amplitudes in order to produce the optimal electrical power.

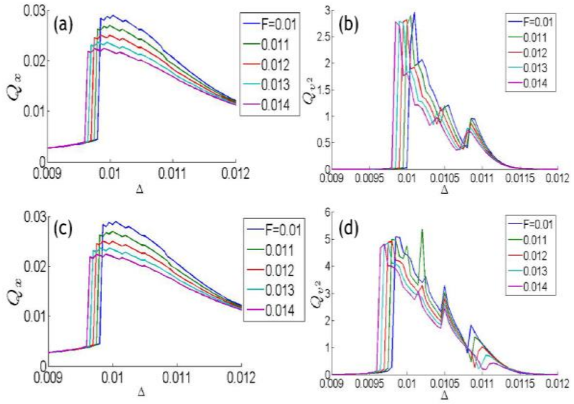
Keywords: Nonlinear dynamics; optimization; vibrational resonance; energy harvesting.

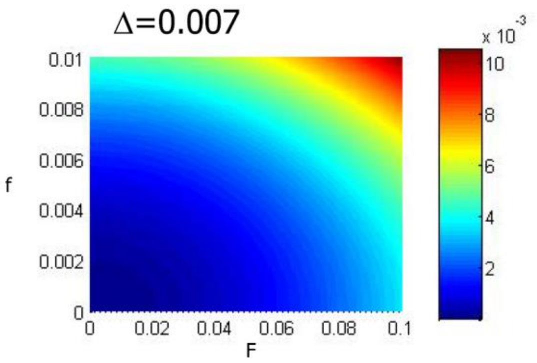
1. Introduction

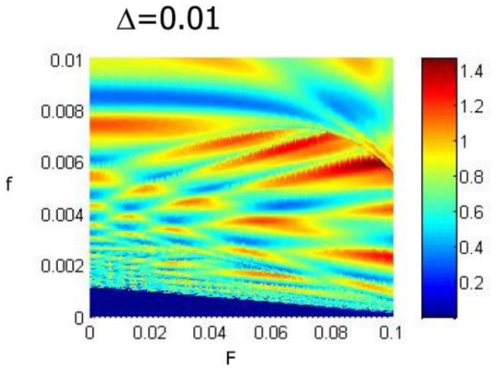
It is possible, all around us, to see small but powerful electrical devices greedy for energy. In fact, the

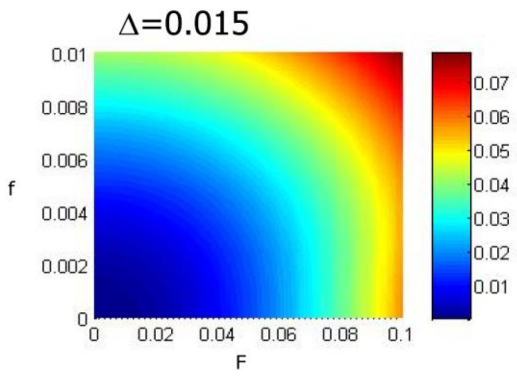
has grown as an interesting open research field, called *energy harvesting*.

In this sense, mechanical vibrations are a possible and reliable energy source that can be exploited.









La propiedad típica de un Sistema con retraso es que la evolución futura del sistema no depende solo de su estado presente sino que de su historia

$$\dot{x} = f(x, x_\tau)$$
$$x_\tau = x(t - \tau)$$

Duffing sobreamortiguado con retraso

$$\dot{x} = x + x_\tau - (1 + \alpha)x^3$$

Esta propiedad puede transformar un sistema con dinámicas sencillas en uno con dinámicas más complejas.

Bogdanov-Takens resonance in ENSO like oscilator

$$\dot{x} = \alpha x_\tau + x - (1+\alpha)x^3 + F\sin\Omega t$$
$$x_\tau = x(t-\tau)$$

Nonlinear Dyn (2018) 91:1939–1947
https://doi.org/10.1007/s11071-017-3992-1

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ORIGINAL PAPER

Bogdanov–Takens resonance in time-delayed systems

Mattia Coccolo · BeiBei Zhu ·
Miguel A.F. Sanjuán · Jesús M. Sanz-Serna

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Abstract We analyze the oscillatory dynamics of a time-delayed dynamical system subjected to a periodic external forcing. We show that, for certain values of the delay, the response can be greatly enhanced by a *very small forcing amplitude*. This phenomenon is related to the presence of a *Bogdanov–Takens* bifurcation and displays some analogies to other resonance phenomena, but also substantial differences.

Keywords Nonlinear oscillations · Delay systems · Resonance

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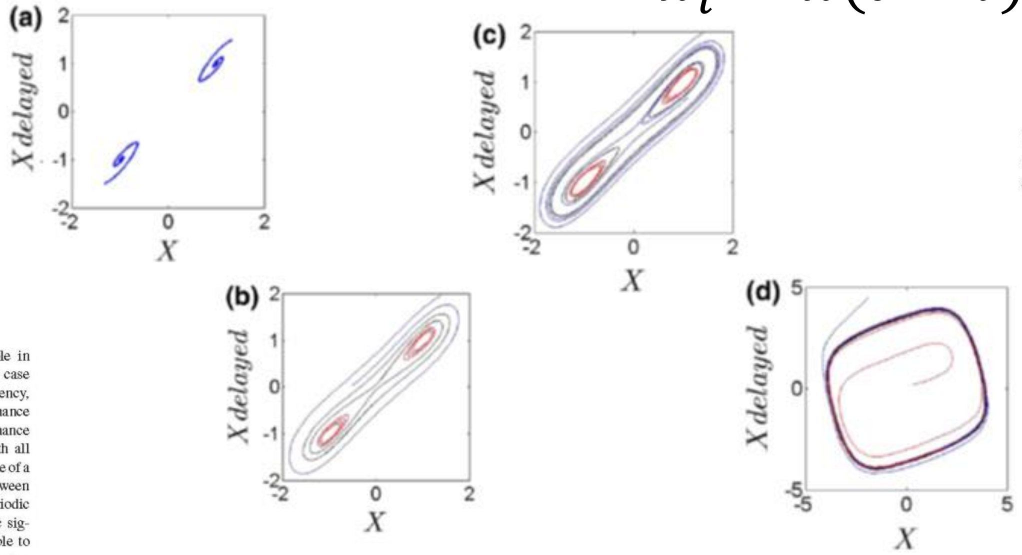
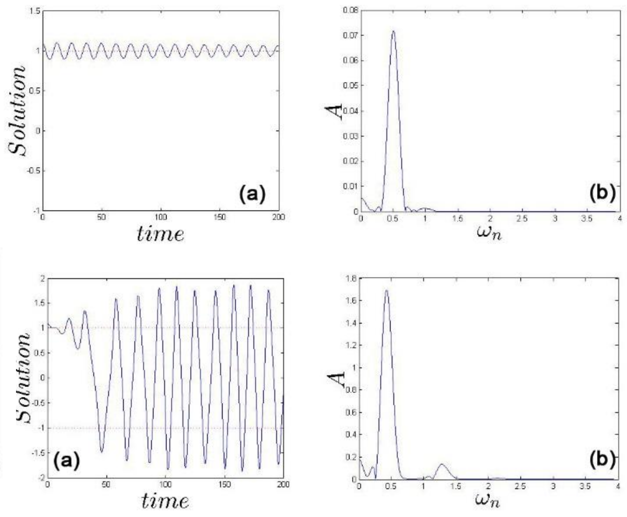
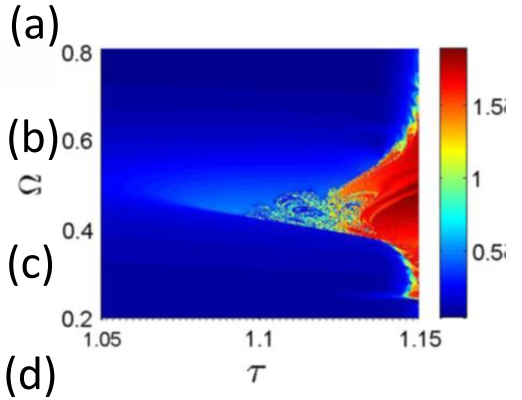
1 Introduction

Different resonance phenomena play a key role in the sciences. Examples, beyond the simplest case of a linear system forced at its natural frequency, include stochastic resonance [1,2], chaotic resonance [3], coherence resonance [4] and vibrational resonance (VR) [5]. For a recent monograph dealing with all these phenomena, see [6]. The stochastic resonance of a bistable system is triggered by the cooperation between noise and a weak periodic forcing, or even an aperiodic forcing. The noise can be replaced by a chaotic signal to obtain chaotic resonance. It is also possible to have noise-induced resonance in the absence of external periodic forces, a phenomenon called coherence resonance. A nonlinear system driven by a biharmonic forcing, with a frequency faster than the other, can show VR. Resonances appear not only in systems described by ordinary differential equations, but also in time-delayed systems. Time-delay effects arise frequently in practical problems and have received much attention in recent years [7–11]. Hereditary effects are sometimes unavoidable and may easily turn a well-behaved system into one displaying very complex dynamics. A simple example is provided by Gumowski and Mira [12], who demonstrate that the presence of delays may destroy stability and cause periodic oscillations in systems governed by differential equations. Vibrational resonance occurs in time-delayed systems with two harmonic forcings of different frequencies [13–16]. Furthermore, delay systems often possess oscillatory behavior even





Solutions as a function of τ for fixed α. The Bogdanov – Takens bifurcation	
$\tau < \tau_c$	The equilibrium points ± 1 attract (most) solutions.
Periodic saddle–node bifurcation at τ_c. A stable loop L_s and a smaller unstable loop L_u are born.	
$\tau_c < \tau < \tau_h$	The equilibrium points attract solutions inside L_u. Outside L_u solutions are attracted to L_s.
Homoclinic bifurcation at $\tau = \tau_h$. The loop L_u gives rise to two unstable loops $L_{\pm 1}$ around ± 1 respectively.	
$\tau_h < \tau < \tau_0$	Solutions inside $L_{\pm 1}$ attracted to corresponding equilibrium. Other solutions are attracted to L_s.
Hopf bifurcation at $\tau = \tau_0$. The loops $L_{\pm 1}$ merge with the corresponding equilibrium.	
Large τ	L_s attracts most solutions.



Delay induced resonance

International Journal of Bifurcation and Chaos

Delay-Induced Resonance in the Time-Delayed Duffing Oscillator

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Received (to be inserted by publisher)

The phenomenon of delay-induced resonance implies that in a nonlinear system a time-delay term may be used as an effective enhancer of the oscillations caused by an external forcing maintaining the same frequency. This is possible for the paramaters for which the time-delay induces sustained oscillations. Here, we study this type of resonance in the overdamped and underdamped time-delayed Duffing oscillator, and we explore some new features. One of them is the conjugate phenomenon: the oscillations caused by the time-delay may be enhanced by means of the forcing without modifying their frequency. The resonance takes place when the frequency of the oscillations induced by the time-delay matches the ones caused by the forcing and vice versa. This is an interesting result as the nature of both perturbations is different. Even for the parameters for which the time-delay does not induce sustained oscillations, we show that a resonance may appear following a different mechanism.

Keywords: Bifurcation analysis, Duffing oscillator, Resonance, Delay, Delay-induced resonance.

Auguri!!

Feliz 60 vueltas a nuestra estrella