Universidad Rey Juan Carlos

Dispersión caótica: Ruido y Relatividad

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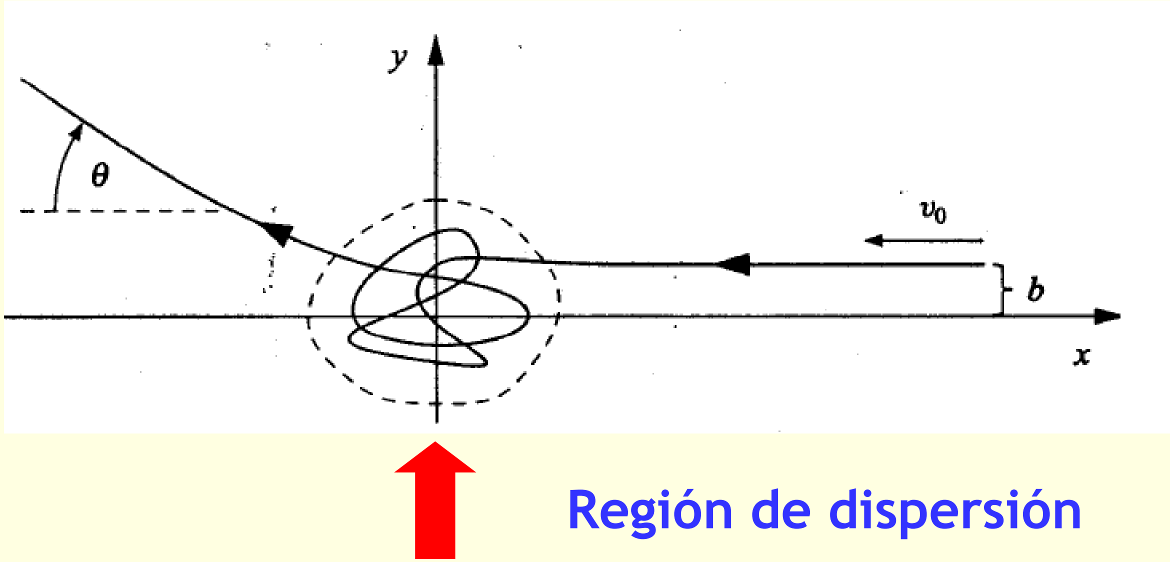
Jornada Científica en Homenaje
al Prof. Miguel Ángel Fernández Sanjuan
por su 60 cumpleaños

12 de diciembre de 2019

INTRODUCCIÓN

- **Dispersión caótica:** estudia la relación entre una variable de entrada y una variable de salida.
  - **Variable de entrada:** caracteriza la condición inicial.
  - **Variable de salida:** caracteriza el estado final del sistema.



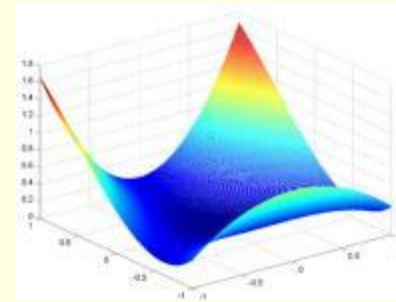
Región de dispersión

- La dispersión caótica es una manifestación física de caos transitorio.

INTRODUCCIÓN

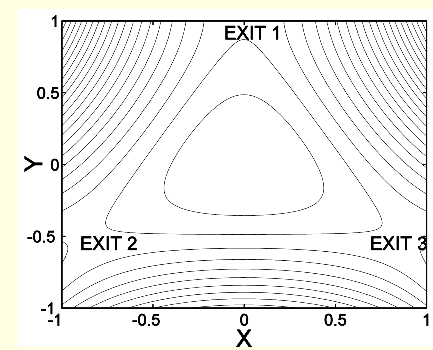
Hamiltoniano Hénon-Heiles (1964) (I):

$$H = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) + \frac{1}{2}\left(x^2 + y^2\right) + x^2 y - \frac{1}{3} y^3$$



El sistema presenta 3 salidas a través de las cuales las partículas pueden escapar.

Hay un límite de energía que define el umbral a partir del cual las partículas pueden escapar E=1/6 (régimen no-hyperbolico).



1. WEAKLY NOISY CHAOTIC SCATTERING (2013)

PHYSICAL REVIEW E **88**, 032914 (2013)

Weakly noisy chaotic scattering

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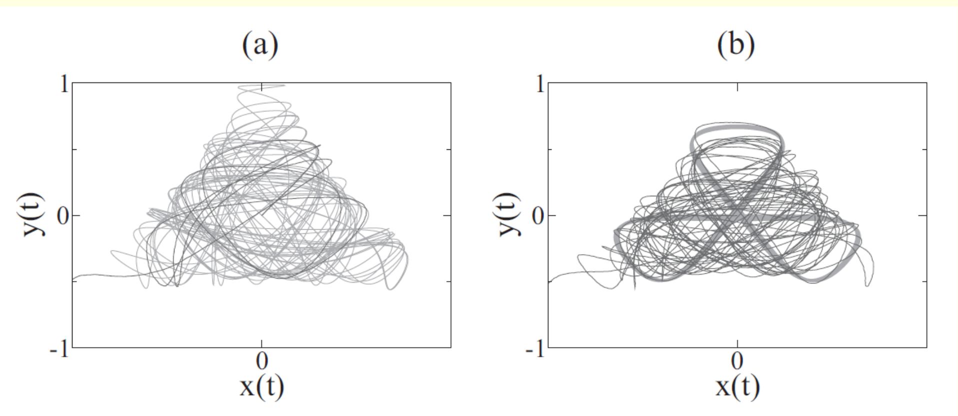
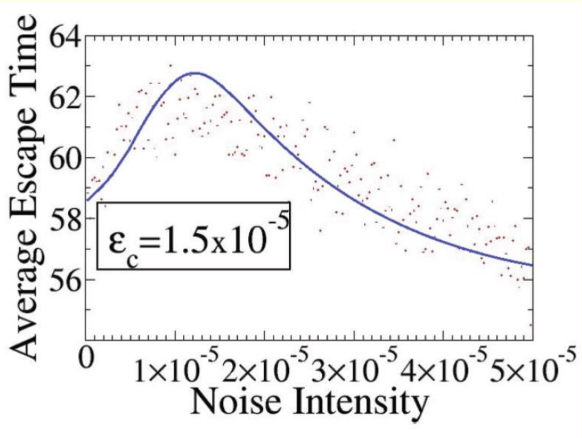
(Received 30 April 2013; published 23 September 2013)

The effect of a weak source of noise on the chaotic scattering is relevant to situations of physical interest. We investigate how a weak source of additive uncorrelated Gaussian noise affects both the dynamics and the topology of a paradigmatic chaotic scattering problem as the one taking place in the open nonhyperbolic regime of the Hénon-Heiles Hamiltonian system. We have found long transients for the time escape distributions for critical values of the noise intensity for which the particles escape slower as compared with the noiseless case. An analysis of the survival probability of the scattering function versus the Gaussian noise intensity shows a smooth curve with one local maximum and with one local minimum which are related to those long transients and with the basin structure in phase space. On the other hand, the computation of the exit basins in phase space shows a quadratic curve for which the basin boundaries lose their fractal-like structure as noise turned on.

DOI: 10.1103/PhysRevE.88.032914 PACS number(s): 05.45.Ac, 05.45.Df, 05.45.Pq





2. GLOBAL RELATIVISTIC EFFECTS IN CHAOTIC SCATTERING (2017)

RESULTADOS ANTERIORES

PHYSICAL REVIEW E **95**, 032205 (2017)

Global relativistic effects in chaotic scattering

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(Received 24 November 2016; published 8 March 2017)

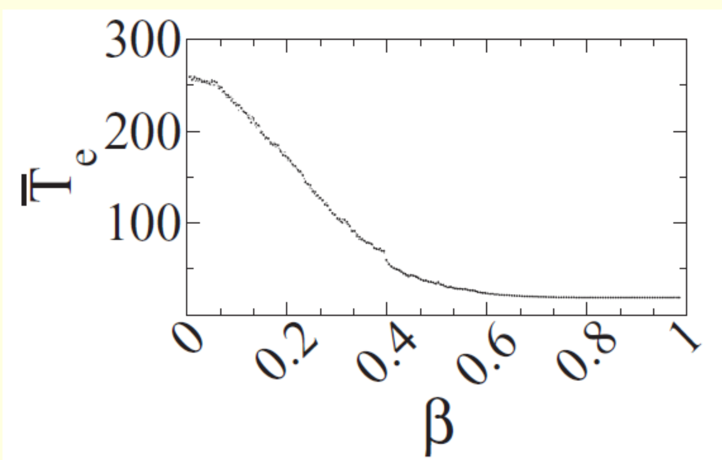
The phenomenon of chaotic scattering is very relevant in different fields of science and engineering. It has been mainly studied in the context of Newtonian mechanics, where the velocities of the particles are low in comparison with the speed of light. Here, we analyze global properties such as the escape time distribution and the decay law of the Hénon-Heiles system in the context of special relativity. Our results show that the average escape time decreases with increasing values of the relativistic factor β. As a matter of fact, we have found a crossover point for which the KAM islands in the phase space are destroyed when $\beta \simeq 0.4$. On the other hand, the study of the survival probability of particles in the scattering region shows an algebraic decay for values of $\beta \leqslant 0.4$, and this law becomes exponential for $\beta > 0.4$. Surprisingly, a scaling law between the exponent of the decay law and the β factor is uncovered where a quadratic fitting between them is found. The results of our numerical simulations agree faithfully with our qualitative arguments. We expect this work to be useful for a better understanding of both chaotic and relativistic systems.

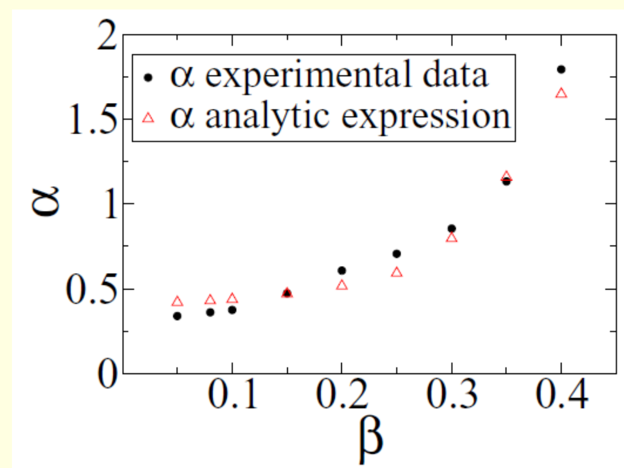
DOI: 10.1103/PhysRevE.95.032205

$$\dot{\mathbf{p}} = -\frac{\partial H}{\partial \mathbf{r}} = -\nabla V(\mathbf{r}),$$

$$\dot{\mathbf{r}} = \mathbf{v} = \frac{\partial H}{\partial \mathbf{p}} = \frac{\mathbf{p}}{m\gamma},$$

$$\gamma = \sqrt{1 + \frac{\mathbf{p}^2}{m^2 c^2}} = \frac{1}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}}.$$





3. UNCERTAINTY DIMENSION AND BASIN ENTROPY IN CHAOTIC SCATTERING (2018)

RESULTADOS ANTERIORES

PHYSICAL REVIEW E **97**, 042214 (2018)

Uncertainty dimension and basin entropy in relativistic chaotic scattering

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(Received 9 March 2018; published 20 April 2018)

Chaotic scattering is an important topic in nonlinear dynamics and chaos with applications in several fields in physics and engineering. The study of this phenomenon in relativistic systems has received little attention as compared to the Newtonian case. Here we focus our work on the study of some relevant characteristics of the exit basin topology in the relativistic Hénon-Heiles system: the uncertainty dimension, the Wada property, and the basin entropy. Our main findings for the uncertainty dimension show two different behaviors insofar as we change the relativistic parameter β, in which a crossover behavior is uncovered. This crossover point is related with the disappearance of KAM islands in phase space, which happens for velocity values above the ultrarelativistic limit, $v > 0.1c$. This result is supported by numerical simulations and by qualitative analysis, which are in good agreement. On the other hand, the computation of the exit basins in the phase space suggests the existence of Wada basins for a range of $\beta < 0.625$. We also studied the evolution of the exit basins in a quantitative manner by computing the basin entropy, which shows a maximum value for $\beta \approx 0.2$. This last quantity is related to the uncertainty in the prediction of the final fate of the system. Finally, our work is relevant in galactic dynamics, and it also has important implications in other topics in physics such as as in the Störmer problem, among others.

DOI: 10.1103/PhysRevE.97.042214

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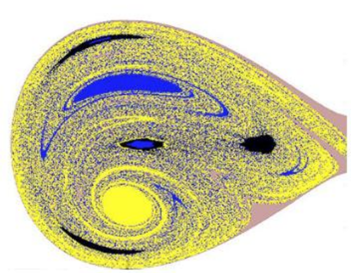
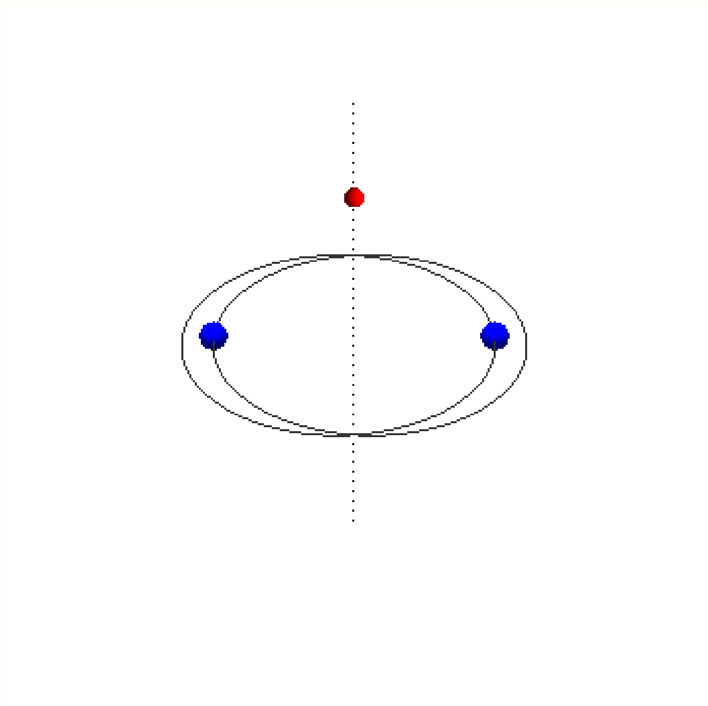


From the article:
Uncertainty dimension and basin entropy in relativistic chaotic scattering
Juan D. Bernal, Jesús M. Seoane, and Miguel A. F. Sanjuán
Phys. Rev. E **97**, 042214 (2018)

4. CHAOTIC SCATTERING IN THE RELATIVISTIC SITNIKOV PROBLEM (esp. 2020)

TRBAJO ACTUAL

- Estudiar la dispersión caótica en el **problema de Sitnikov relativista**.

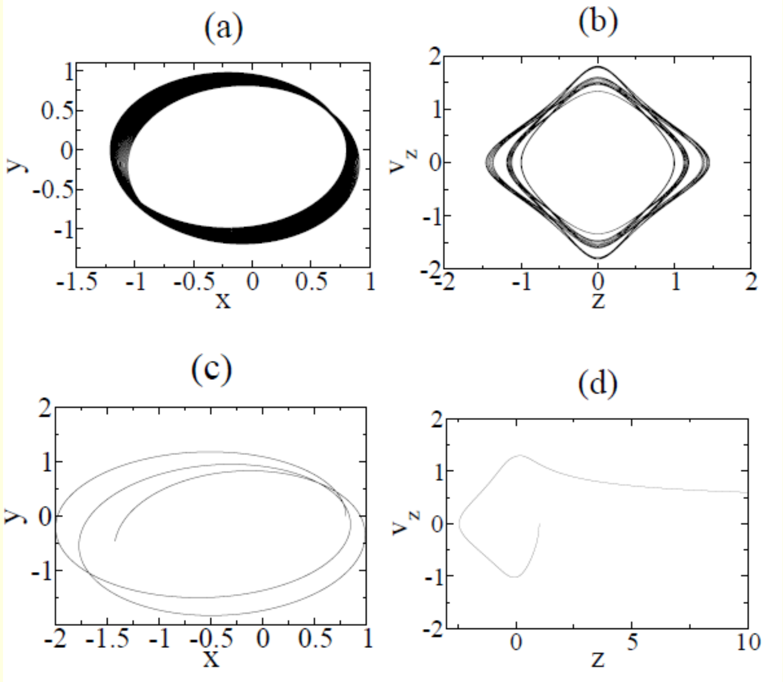


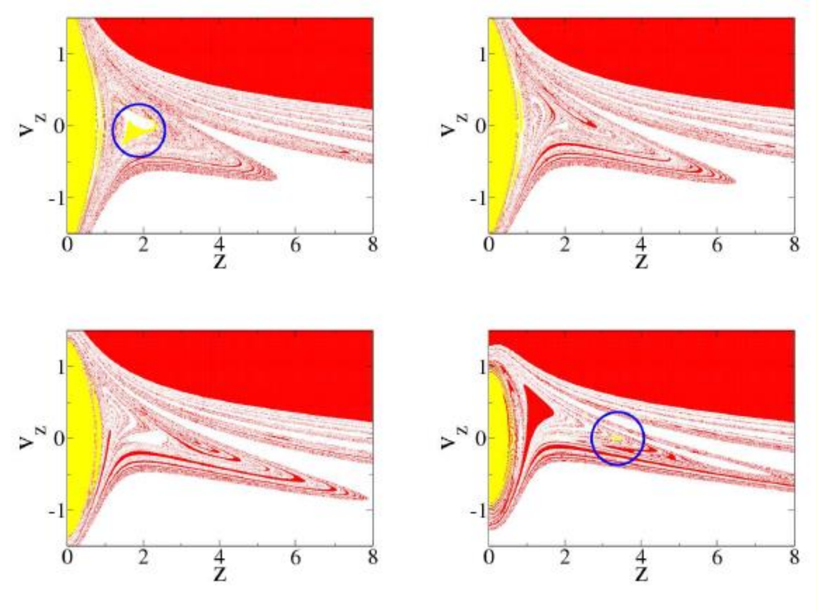
Propuesta futura publicación:

Montly Notices of the **Royal Astronomical Society**.

Colaboración con **Prof. Lian Huang** (Catedrático de la Lanzhou University, School of Physical Science and Technology, China) y con **Juan Carlos Vallejo Chavarino** (profesor asociado de la URJC y miembro del grupo AEGORA de la UCM).

4. CHAOTIC SCATTERING IN THE RELATIVISTIC SITNIKOV PROBLEM (esp. 2020)





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¡Gracias!

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