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ESSAY REVIEW

Modern Dynamics

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A review of **Introduction to Modern Dynamics: Chaos, Networks, Space and Time** by David D. Nolte, Oxford, Oxford University Press, 2014, 448 pp., £32.50 (paperback), ISBN 978-0-19-965704-9. Scope: textbook. Level: advanced undergraduate, post graduate students.

Imagine a textbook whose contents are: Geometric Mechanics, Nonlinear Dynamics, Complex Systems, and Relativity and Spacetime. A priori, a first question might arise: Is not this a kind of potpourri of subjects all of them mixed together? Is there a thread that could give a sense to these subjects apparently disconnected? Well, perhaps the reader might ask himself first what are the specific contents of these four different subjects. Of course, a textbook has an author and the author decides what kind of information to include. What I mean is that distinct authors would certainly include different contents related to the previous subjects depending on their taste, knowledge or objectives. In this case, the author has decided to include Physics and Geometry, and Hamiltonian Dynamics and Phase Space as two chapters of the Geometric Mechanics part. In the second part corresponding to Nonlinear Dynamics, three chapters: Nonlinear Dynamics and Chaos, Coupled Oscillators and Synchronization, and Network Dynamics. The choices for the third part on Complex Systems could be numerous, and of a different nature. The author has included three chapters: Neurodynamics and Neural Networks, Evolutionary Dynamics and Economic Dynamics. And finally for the fourth part on Relativity and Spacetime, there are also three chapters: Metric Spaces and Geodesic motion, that is, some sort of review on elementary differential geometry, Relativistic Dynamics, where the Lorentz transformations and the Special Theory of Relativity are described and finally The General Theory of Relativity and Gravitation.

Again one needs to ask whether there is a thread giving sense to all this information. For some readers, it might be a difficult task to answer this. However, in my opinion, a simple response to this question is just *dynamics*, in particular, trajectories in different kinds of spaces. In other words, geometry and dynamics.

Traditionally, the idea of dynamics in textbooks has evolved with time. When we think about textbooks used for the teaching of mechanics in the last century, we may think on the book *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies* by Edmund T. Whittaker, Cambridge University Press (1st edition), 1904; *Principles of Mechanics* by John L. Synge and Byron A. Griffith, McGraw-Hill (1st edition), 1942 and *Classical Mechanics* by Herbert Goldstein, Addison-Wesley (1st edition), 1950. The three of them are very different in style and contents. The objective of the Goldstein book is to give the students the basic knowledge of mechanics with the goal of bringing them into the classical field theories, perhaps with the idea to provide them with the necessary tools to face quantum field theory.

As a sign that the contents of a classical subject like dynamics changes with time, it is worth to mention that before the death of Herbert Goldstein in 2005, a new third edition of the book was released, with the collaboration of Charles P. Poole and John L. Safko, *Classical Mechanics* by Herbert Goldstein, Charles P. Poole, John L. Safko (3rd edition) (2001), Addison-Wesley. In this third edition, new material was introduced and some parts were reformulated, but perhaps the main innovation was to introduce a new chapter on classical chaos, and as the authors write in the preface to the third edition:

> The chapter on Chaos is a necessary addition because of the current interest in nonlinear dynamics which has begun to play a significant role in applications of classical dynamics. The majority of classical mechanics problems and applications in the real world include nonlinearities, and it is important for the student to have a grasp of the complexities involved, and of the new properties that can emerge. It is also important to realize the role of fractal dimensionality in chaos.

Presently, when we talk about dynamics, we understand any kind of motion of a variable, or set of variables. From this new point of view, we may encounter dynamics everywhere, virtually in every field of science. There has been a kind of transition from an idea of dynamics mainly considering physical and mechanical bodies to an idea of dynamics including all kinds of phenomena that change with time. There is certainly motion of celestial bodies, of mechanical systems, such as solid structures, spinning tops, beams, but also there is motion of atomic and subatomic particles, motion of the stock market, motion of concentrations in chemical reactions, motion of any physiological variable in biology and medicine, motion of the action potential of neurons, etc.

All this justifies the need to teach this new viewpoint to the future generations of students. In addition, for this noble purpose, new textbooks are needed. With this in mind, the textbook *Introduction to Modern Dynamics: Chaos, Networks, Space and Time* by David D. Nolte constitutes a new perspective of the teaching of dynamics. As mentioned earlier, the main thread in this book is the study of trajectories in different spaces where motion takes place. This is the reason why the author has considered basically continuous dynamical systems or flows. Since the goal of the book is to offer a kind of unified vision of dynamics including new topics and disciplines, one could argue why certain important aspects of dynamics, such as fluid dynamics, do not appear. Perhaps, it could have been included as another part of the book, but certainly this would have made it too broad and maybe unpractical.

What is really new in this case with respect to some previous textbooks on dynamics? Mainly parts 2 and 3, where nonlinear dynamics, chaos and complex systems are explained, since parts 1 and 4 are usually covered in standard physics undergraduate courses on classical mechanics, relativity and gravitation. In this sense, one may also argue why this textbook is necessary when there are wonderful textbooks on Nonlinear Dynamics and Chaos such as *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering* by Steven H. Strogatz, Perseus Books, (1st edition) 1994 and *Chaos – An Introduction to Dynamical Systems* by Kathleen T. Alligood, Tim D. Sauer and James A. Yorke, Springer (1st edition), 1997. Most of the information offered in parts 2 and 3, though not all, is included in the previous references, though the Strogatz book is more devoted to applications, while the Alligood-Sauer-Yorke is more mathematically oriented. However, what in my opinion makes Nolte's book special is that 'The traditional topics of mechanics are integrated into the broader view of modern dynamics that draws from the theory of complex systems' in words of the author. And what I consider distinctive is the attempt to integrate this new vision of dynamics into the core education of a physicist. Since despite the interdisciplinary nature of some of these topics, I think that they belong to the tradition of physics and as such should be transmitted to the future generations of physics students.

I really believe this new textbook will be a useful tool and a complement to help the students to open their minds into this broader vision of dynamics. Furthermore, this will be very helpful for the students to face new challenges in topical issues in the world of physics research.

As the author mentions in the preface: 'The best parts of physics are the last topics that our students ever see'. He also writes:

> The structure of this book combines the three main topics of modern dynamics—chaos theory, dynamics on complex networks, and the geometry of dynamical spaces—into a coherent framework. By taking a geometric view of physics, concentrating on the time evolution of physical systems as trajectories through abstract spaces, these topics share a common and simple mathematical language with which any student can gain a unified physical intuition. Given the growing importance of complex dynamical systems in many areas of science and technology, this text provides students with an up-to-date foundation for their future careers.

David Nolte also tell us 'All changing systems, whether in biology or economics or computer science or photons in orbit around a black hole, are understood as trajectories in abstract dynamical spaces'.

By following the thread of studying trajectories in dynamical spaces, the author is able to establish a wide range of complex behaviour in dynamical systems of different nature, by simply analysing the associated flows that provide many intuitive insights.

If we discard the basic topics mentioned earlier, there are still a series of special topics covered in this textbook mainly devoted to nonlinear dynamics, complex systems and relativity of spacetime.

Chaos theory is one of the fundamental chapters since it provides the basic tools of nonlinear dynamics necessary for understanding some important subjects. Furthermore, it also constitutes the *language* of most of the contents related to complex systems. Simple and important concepts such as the Poincaré section and strange attractors are explained, including basic notions of fractal geometry, and discrete dynamical systems or iterative maps.

One important aspect of complex systems is holism, that is, the idea that the whole is greater than the sum of its parts, and the properties that emerge out of the interactions among its parts. Consequently, to learn the basics of coupled systems is fundamental. For this purpose, there is first a simple description of coupled linear oscillators, which is a rather standard topic in classical mechanics. Later, coupled nonlinear oscillators are discussed, where the important phenomenon of synchronisation of two or more oscillators, which is a key for an appropriate understanding of other complex systems, is emphasised. Furthermore, to study synchronisation of nonlinear oscillators is perhaps the simplest case for a course inspired in basic knowledge in classical mechanics.

But not only the couplings are important, rather how they are made, what bring us to network theory, and to the study of some basic classes of networks such as small-world networks and scale-free networks. This is a topic of strong current research activity, which in many instances has gone much beyond the field of physics. However, the main ideas can be useful for the understanding neurodynamics, evolutionary dynamics and economic dynamics. An interesting mention is given to synchronisation of Poincaré phase oscillators, also called Kuramoto oscillators, which can induce a transition to complete synchronicity.

Complex systems as a field are becoming more relevant not only in physics, but also in other related disciplines, such as neurodynamics. In fact, many consider the brain as the most complex system in the universe. A basic knowledge of the dynamics of a single neuron, such as the Hodgkin–Huxley neuronal model and the much simpler Fitzhugh–Nagumo model, with limit cycle oscillator solutions showing bistability, bifurcations and even chaos is a good starting point. Later, simple neural networks are discussed, including some more complex networks of neurons and their functional properties.

A very active domain where simple ideas of dynamics play an important role is evolutionary dynamics. First of all, it is worth mentioning that some of the earliest explorations into nonlinear dynamics came from studies of population dynamics, such as the logistic equation first introduced in 1838 by the Belgian mathematician Pierre François Verhulst (1804–1849). The great work on evolutionary dynamics and quasi-species carried out by the 1967 Chemistry Nobel Laureate Manfred Eigen and other ideas related to topics such as viral mutation and spread, as well as the evolution of species within a fitness landscape is also discussed.

Another interesting application of the ideas of nonlinear dynamics and complex systems is Economic Dynamics. The author also uses the term Econophysics, as some physicists do, though this last term is perhaps more appropriate in relation to the application of statistical physics ideas to economy. Simple ideas taken from nonlinear dynamics are applied to the dynamics of microeconomics and macroeconomics, showing clear insights about business cycles and the diffusion of prices on the stock market. The key factor is to grasp that some ideas of economics can become understandable using concepts from nonlinear dynamics, evolutionary dynamics or network dynamics, all of them treated in the book.

Finally, relativity and spacetime are discussed. One of the first objectives is to give out the basic ideas of differential geometry and geodesic motion as the necessary mathematical tools for the successive chapters on relativity theory. The geometry of space is fully defined in terms of a metric tensor, and trajectories through a dynamical space are discovered to be paths of force-free motion. Furthermore, the geodesic equation is described.

A good and brief synthesis of special relativity is given, by describing the Lorentz transformations, mass–energy equivalence and Minkowski spacetime whose geometric properties are defined by the Minkowski metric. Furthermore, a brief discussion on relativistic forces and noninertial frames connect to the final chapter that generalises all relativistic behaviour.

Finally, the basics of general relativity and gravitation are described. One of the key ideas of this book, the role played by geometry of space on the trajectories, is fundamental for the physics of gravitation. Actually, this geometric view of physics is the responsible for the role played by the force-free geodesics through warped spacetime. Furthermore, Mercury's orbit around the Sun, and trajectories of light past black holes, are simply geodesic flows whose properties are easily understood using simple ideas from nonlinear dynamics.

Every chapter is adorned with small notes, aside of the main text, providing very interesting historical and biographical data on scientists that have contributed to special concepts discussed in the text or just as a way to make much clearer some new ideas and concepts. Another nice point of the book is that at the end of each chapter a summary, a bibliography and a list of homework exercises and in some chapters computational projects are included, taking the reader to a web page where to find the Matlab programs. Thus, the summary provides a succinct relation of the key concepts discussed in the chapter. The bibliography offers a superb collection of key references where one could eventually improve the contents that by nature are synthesised. The books selected are amongst the best I know. Finally, the collection of homework exercises and computational projects are of particular interest for the topics related to nonlinear dynamics, chaos and complex systems.

The book has a website (www.works.bepress.com/ddnolte) where an Appendix and additional support material can be found. The Appendix offers details for some of the mathematical techniques used throughout the textbook, and furthermore a list of Matlab programming examples for specific homework problems among the computational projects. An exceptional document with historical notes and biographies of some of the main contributors to modern dynamics is also included.

Another important issue, not yet discussed, is the level of mathematics required to follow the textbook, which necessarily needs to be taken into account when designing a study course. The textbook clearly emphasises the main concepts and ideas. It is rather synthetic in many issues, which may constitute a strength, but at the same time it requires a set of mathematical tools to follow certain parts. This includes basically, ordinary differential equations, differential geometry and tensor calculus, graph theory and other traditional mathematical methods for physics.

This textbook is designed for undergraduates, which I think is the intention of the author. Nevertheless, it could also be used for graduate courses. It can be considered a non-standard text according to some classical references. That is, most textbooks in classical mechanics and dynamics do not consider many of the subjects treated here. Therefore, a fundamental question is: how to best use this textbook? If we use it for a formal course for undergraduates, to which main subject corresponds? Which level or year? All these might be questions with different responses for different university institutions and systems. The author is a professor of Physics at Purdue University and in his opinion, all the traditional topics of junior-level physics (third year out of four in the US university system) are included, mainly in part 1, some topics in part 2 and special relativity in part 4. So that, in the mind of the author: 'The goal of this textbook is to modernize the teaching of junior-level dynamics, responsive to a changing employment landscape, while retaining the core traditions and common language of dynamics texts' after having added the new topics on complex dynamical systems.

This poses in my opinion one of the most critical aspects to this book. Despite all its virtues that I have praised and described along this essay, I find that in some circumstances it might not be easy to use it for a standard undergraduate course, due to the rigidity of some university systems. The author seems to be fully aware of this problem, so that he dedicates almost four pages at the very beginning to justify and explain how to give a good use of this new approach. I cannot but congratulate the author for this intelligent though risky proposal that I understand is intended as an incentive to improve and modernise the teaching of dynamics.

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