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Synchronization of uncoupled excitable systems induced by white and coloured noise

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Abstract. We study, both numerically and experimentally, the synchronization of uncoupled excitable systems due to a common noise. We consider two identical FitzHugh–Nagumo systems, which display both spiking and non-spiking behaviours in chaotic or periodic regimes. An electronic circuit provides a laboratory implementation of these dynamics. Synchronization is tested with both white and coloured noise, showing that coloured noise is more effective in inducing synchronization of the systems. We also study the effects on the synchronization of parameter mismatch and of the presence of intrinsic (not common) noise, and we conclude that the best performance of coloured noise is robust under these distortions.

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Contents

1. Introduction

The FitzHugh–Nagumo (FHN) system [1] has been broadly investigated in the past. In spite of its mathematical simplicity, many of the characteristics observed in neuronal cells are reproduced by this system, which is one of the systems that is most utilized to study the spiking activity of a neuron [2]. In particular, the periodically driven FHN system has been used to investigate the effects of external perturbations on the generation of electrical pulses in neurons, as well as the role of noise in the encoding process [3]. The FHN system has become the paradigm of an excitable system in which to explore control methods enabling us to tame or enhance the spiking dynamics [4].

On the other hand, the study of synchronization phenomena has many applications in different branches of science [5, 6]. Following the pioneering work of Pecora and Carroll [7], several types of synchronization phenomena have been observed. In particular, the phenomenon of common noise synchronization has received great attention. From a theoretical point of view, such a problem is related to global convergence of trajectories in an ensemble of random dynamical systems, that is, the presence of a negative leading Lyapunov exponent, which is an indicator of synchrony within the ensemble (see [8–10]). From a more physical point of view, such an issue received great attention too, and it has been discussed in several works, using models displaying both limit cycles and chaotic attractors [10–16].

Resonance effects on neural spike time reliability [17] and synchronization in uncoupled systems due to a common input in neuronal dynamics have been observed in neuroscience experiments [18, 19]. Different investigations with a common noise have been carried out on a pair of integrate-and-Fire systems (IF) [20, 21] and on a single IF neuron model subjected to a broad-band noise containing a frequency resonance [17]. However, in such cases there is no chaos to start with (indeed, in IF systems, chaos implies mutual coupling of different individuals [22]).

In this paper, we aim to improve the understanding of the interactions between the coloured noise correlation time and the intrinsic excitable chaotic condition of the noiseless system in common noise synchronization; thus we revisit a chaotic situation. However, at variance with the previous approaches [10, 13–16] we also implement a laboratory circuitry, in order to compare numerical and experimental results. Due to the widespread use of the FHN models in neuron dynamics, we consider here these systems as a paradigm, making as well a laboratory implementation of two identical uncoupled excitable FHN systems [1].

Considering this, the basic setting that we use in order to explore the common noise synchronization throughout this paper consists of two FHN systems subjected to both a common periodic driving and a common noise, which can be either white or coloured.

In this context, the main goal of this paper is to provide evidence that chaotic synchronization induced by noise in uncoupled excitable systems is better realized by coloured (Ornstein–Uhlenbeck) noise than by white noise. Although it has recently been shown that the onset of synchronization depends on the type of chosen noise [23], we can give a heuristic argument explaining this fact based on the following considerations. A driven FHN model displays low-amplitude oscillations due to the forcing term. Whenever the induced oscillation amplitude overcomes the excitability threshold, a large isolated spike emerges. Chaotic behaviour in the FHN system is mainly the result of the erratic distribution of interspike intervals. If we look at the chaotic power spectrum, we have a broad spectrum around the sharp peak of the forcing. In order to get synchronization, one must deal with the interspike interval that develops at frequencies below the peak. Thus, if we compare coloured and white noise at the same amplitude, the former has a larger contribution at low frequencies and hence is more effective. Both numerical and experimental evidence of this fact is provided in this paper.

Another important issue that we address in this paper is the effect of a parameter mismatch on this type of synchronization of the two FHN systems. We also evaluate the effect of intrinsic noise on the systems, a noise that is different in each FHN system and uncorrelated with the common noise. All the results are tested in an electronic circuit that captures the dynamics of the two neurons. Notice that since the FHN neuron model is paradigmatic in neuronal dynamics, the results obtained in this work can be, in principle, generalized to higher-dimensional systems as the Hodgkin–Huxley model [24].

The paper is organized as follows. In section 2, we describe the two uncoupled FHN systems with a common noisy input. Section 3 shows numerical evidence of mutual spike synchronization induced by noise. The effects of a parameter mismatch and intrinsic noise in each FHN system are also studied. In section 4, we describe the electronic analogue simulation with the experimental confirmation of noise-induced synchronization. Finally, discussions of the main results and conclusions are presented in section 5.

2. Model description

As we said in the Introduction, we are interested in the common noise synchronization effect in two chaotic excitable systems. For this reason, in this paper we are going to consider two periodically driven chaotic FHN systems. The equations of this type of system are the following:

$$
\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}t} &= c(-y + x - (x^3/3) + S(t)), \\
\frac{\mathrm{d}y}{\mathrm{d}t} &= x - by + a,
\end{aligned}
\tag{1}
$$

where, with reference to neurophysiology, $x(t)$ is called the voltage variable, $y(t)$ the recovery variable, $S(t) = A\sin(\frac{2\pi}{T}t)$ is an external forcing of period T and amplitude A, and a, b and c are real positive constants. By increasing the amplitude of the external forcing, the system undergoes a cascade of period doubling bifurcations and, for sufficiently high values of A, a spiking regime appears. This regime is characterized by the fact that $x(t) > 1$ for different t values. This is shown in the bifurcation diagram of figure 1, where the voltage variable x is represented as a function of A, and the spiking begins at $A \geqslant 0.465$. The parameters used in this bifurcation diagram as well as in the rest of the numerical simulations are $a = 0.7$, $b = 0.8$, $c = 12.5$ and $T = 0.636$.

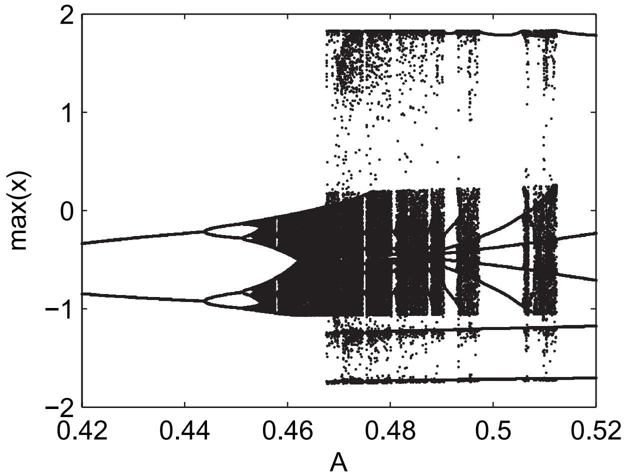

Figure 1. Numerical bifurcation diagram for the voltage variable x by taking as control parameter the amplitude of the external forcing A. The spiking regime is observed for high values of A, whereas the non-spiking regime is observed for low values of A. Parameter values are $a = 0.7$, $b = 0.8$, $c = 12.5$ and $T = 0.636$.

Thus, we can distinguish two different behaviours, spiking and non-spiking [4]. In figures 2(a) and (b), two experimental trajectories in phase space corresponding to the two different regimes are shown. A time series corresponding to the spiking situation is shown in figure 2(c). Notice that in both cases the dynamics are chaotic.

Since our purpose is to study the synchronization induced by common noise in two uncoupled excitable systems, we consider two identical FHN systems with external noise. The equations are

$$\begin{aligned} \frac{\mathrm{d}x_i}{\mathrm{d}t} &= c(-y_i + x_i - (x_i^3/3) + S(t) + \xi(t)), \\ \frac{\mathrm{d}y_i}{\mathrm{d}t} &= x_i - by_i + a, \end{aligned} \tag{2}$$

where $i = 1, 2$, and $\xi(t)$ is a Gaussian random process. Notice that $\xi(t)$ is the same for the two systems; in other words, we have a *common* noise acting on both systems.

For white noise, $\xi(t)$ has the following properties: $\langle \xi(t) \rangle = 0$ and $\langle \xi(t)\xi(s) \rangle = 2D\delta(t - s) = \sigma^2\delta(t - s)$, where D is the noise intensity and σ is its standard deviation. As usual, δ is Dirac's delta function and $\langle \cdots \rangle$ denotes averaging over the realizations of ξ.

In this paper, we also consider the effect of a common Ornstein–Uhlenbeck (coloured) noise [25]. In this situation, we replace $\xi(t)$ in (2) by $\varepsilon(t)$, where the dynamical evolution of $\varepsilon(t)$ is represented by

$$\dot{\varepsilon} = -\gamma\varepsilon(t) + \gamma\xi(t), \tag{3}$$

γ being a positive constant and $\xi(t)$ being a white Gaussian noise with $\sigma = 1$. The coloured noise is an exponentially correlated noise with the following properties: $\langle \varepsilon(t) \rangle = 0$, and $\langle \varepsilon(t)\varepsilon(s) \rangle = D\gamma \exp[-\gamma|t - s|]$. Note that the standard deviation of this type of noise is $\sigma = \sqrt{D\gamma}$, where γ is the inverse of the noise correlation time and D is the intensity of the white noise used in (3).

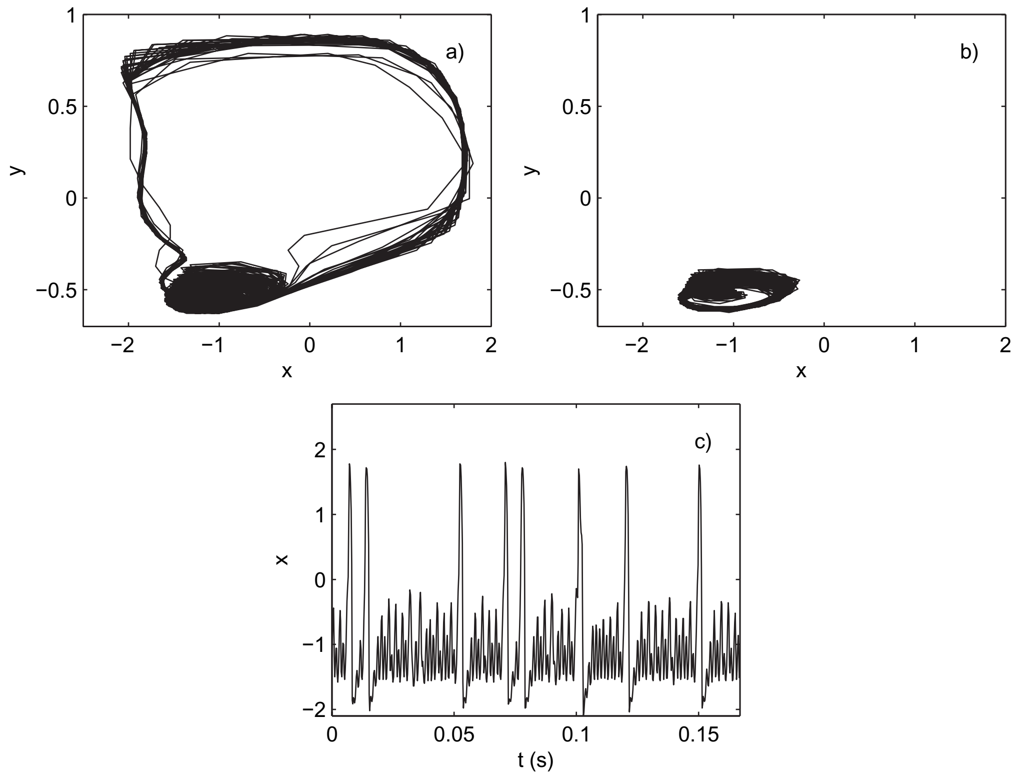

Figure 2. Experimental trajectories in the phase space corresponding to (a) the spiking regime, (b) the non-spiking regime and (c) the time sequence from which (a) is extracted. One can see that the low amplitude between large spikes is mainly a forced limit cycle and is perturbed by the chaotic dynamics.

In order to see more clearly why coloured noise should be expected to enhance more easily the synchronization of the systems, we report in figure 3 the numerical power spectra for the noiseless forced FHN system, figure 3(a), together with power spectra for coloured noise, figure 3(b), and for white noise, figure 3(c) (both noises with the same value of the variance σ). As we explained in the Introduction, the power spectrum of the coloured noise is steeper around frequencies smaller than the frequency of the periodic forcing ($1/T \approx 1.57$), corresponding to spiking frequencies. That is the reason why we should expect coloured noise to be better suited to enhance common noise synchronization.

The next section provides numerical evidence of this fact.

3. Numerical results on noise-induced synchronization

In this section, we study numerically the synchronization induced by noise in two uncoupled FHN systems under two different kinds of noise: white noise and coloured noise. We analyse two different situations: identical systems and systems with a mismatch parameter. In the simulations performed in this section, we use the *Heun algorithm*, which is one of the standard routines for integration of stochastic differential equations [26], using a sampling of 500 integration steps per cycle of the forcing.

In order to have a first insight into the synchronization induced by common noise, we plot in figures 4(a)–(c) $x_1(t)$ against $x_2(t)$ for different values of the common white noise.

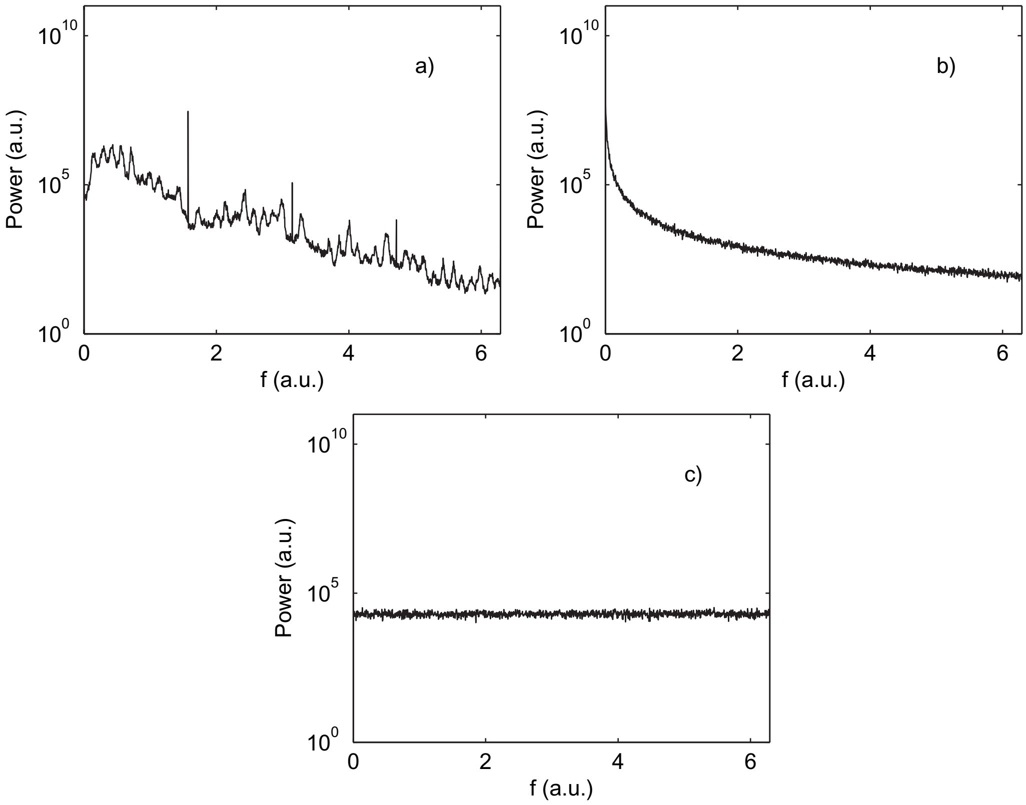

Figure 3. Plots of (a) the power spectrum of the noiseless forced FHN; the main peak is observed at the forcing frequency $1/T$; (b) the power spectrum of the coloured noise whose γ matches the half-width of the spectrum of the noiseless forced FHN; and (c) the power spectrum of white noise. Both spectra ((b) and (c)) correspond to noise with the same σ.

Figure 4(a) represents the noiseless case in which the spikes are not synchronized. By adding a small amount of noise, $\sigma = 0.01$, as in figure 4(b), the synchronization degree of both systems increases, but the systems are not yet fully synchronized. However, for $\sigma = 0.05$, we observe total synchronization between both neurons, as in figure 4(c): $x_1(t) = x_2(t)$ for all t.

Synchronization can be quantified by using different indicators, but here we are going to use a simple indicator that can be easily computed, something that will be useful later when we consider experimental data. This indicator is the synchronization rate R, which we define as the number of synchronized spikes in both neurons in a time series divided by the maximum number of spikes observed in any of the two systems in the same time interval. This is a simple measure that is perfectly suited to the situation that we consider here. Taking into account that we are dealing with two forced FHN oscillators, we can consider that two spikes are synchronized when they coincide within a period T of the external forcing. In this way, there is an intrinsic discretization on the time scale of the dynamics and it is quite natural to assume spike synchronization when two spikes fall inside the same T (it would mean that both systems have responded equally to the common periodic input). Furthermore, simulations show that the shapes of two synchronized spikes are very similar; in particular, their (local) maxima occur for very similar t values, and this feature is preserved when the system is affected by the noise

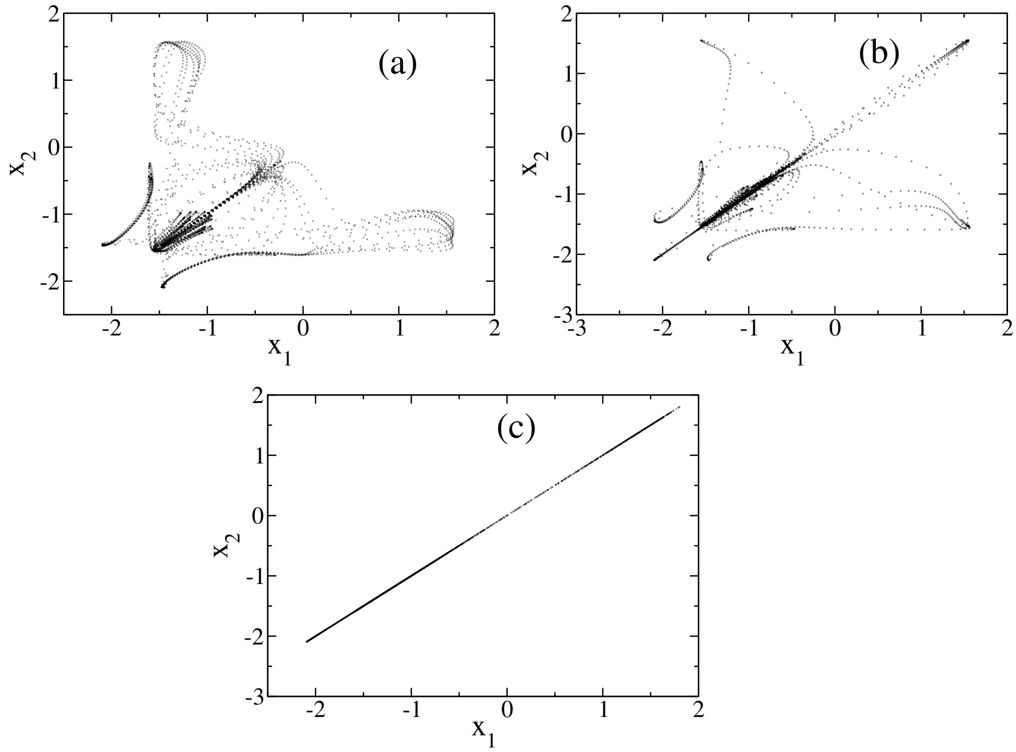

Figure 4. Effect of white noise in the synchronization phenomenon between the two FHN systems. Parameter values are as follows: $A = 0.471$, $a = 0.7$ and $T = 0.636$ and (a) $\sigma = 0$, (b) $\sigma = 0.01$ and (c) $\sigma = 0.05$. The intensity of noise plays a crucial role in the synchronization.

values considered here. In that case, we consider that there has been synchronized spiking. This is illustrated in figure 5, where a plot of x_1 and x_2 against time is shown for a common white noise of $\sigma = 0.02$. We can observe there how some of the spikes of both systems coincide within the same period of the forcing. In this figure, we also observe two synchronized spikes out of four possible ones; so in this case, $R = 0.5$.

In order to estimate the effect of noise in these systems, we have calculated the synchronization rate R for our systems for 50 values of the standard deviation of the common noise σ. We point out that the synchronization rate R observed in a time series strongly depends on the initial conditions in both systems. Thus, in order to observe in a clearer way how R depends on the noise intensity, the values of R shown for each σ are the average values over 100 calculations done with random initial conditions in both systems. For each calculation of R the first 1000 cycles were discarded in order to avoid transient behaviours. The results of our calculation are shown in figure 6, together with the statistical error for some σ values (just some of them are shown for the sake of clarity). We take here (and throughout the paper) the statistical error as three times the standard error of the mean, i.e. $3S(R)/\sqrt{100}$, where $S(R)$ is the standard deviation of the R values obtained. As claimed, we can clearly observe in this figure that, with coloured noise, smaller σ values are needed to have a high synchronization rate R.

Since in realistic situations it is impossible to have two identical systems, we have studied the influence of a parameter mismatch on the synchronization phenomenon. The study of the

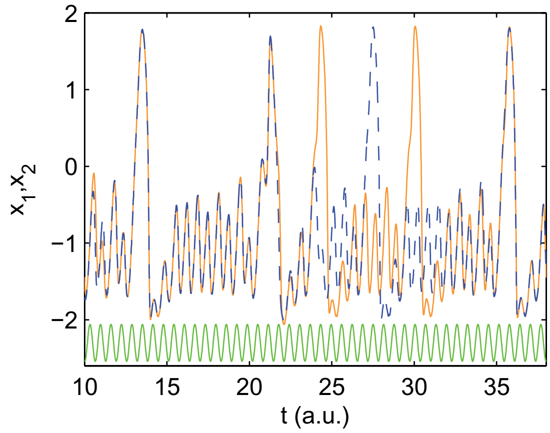

Figure 5. Temporal evolution of the variable x_1 (continuous (orange) grey line) and the variable x_2 (dash-dotted (blue) black line) for identical systems and common white noise with $\sigma = 0.02$. Below them we plot the common forcing ((green) black line). We can observe that when two spikes coincide within a period T of the forcing, they have very similar shapes. Such spikes are considered synchronized. For this short time series, the synchronization rate is $R = 0.5$.

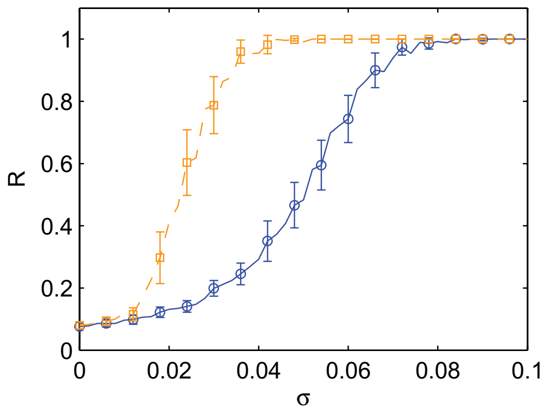

Figure 6. Plot of the synchronization rate R (see text for details) versus the standard deviation of noise for white (continuous (blue) black line, ○) and coloured noise (dashed (orange) grey line, □). Error bars are displayed for one third of the values of σ for the sake of visualization. In both cases, the intensity of the noise produces a positive effect on the synchronization, although clearly at equal σ values there is a higher synchronization degree for coloured noise.

effect of small non-identities (similar to our mismatch) has been performed from an analytical point of view in the context of nonlinear oscillators in the presence of noise [27], using as a prototype model the Van der Pol–Duffing oscillator. Here, in order to study our two FHN models with a parameter mismatch, we have taken different values of the parameter a in the two FHN systems, labelled a_1 for the first FHN system and a_2 for the second one, and computed the synchronization rate R when $a_1 \neq a_2$. In this situation, a decrease in the synchronization of the spikes is observed. The synchronization loss is due to the difference in the spiking frequencies of

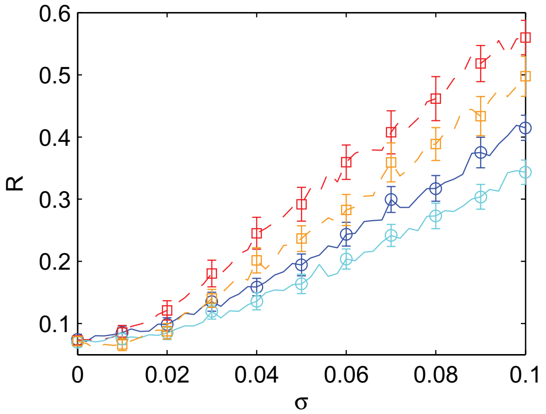

Figure 7. Synchronization rate R as a function of σ for white noise (continuous (blue) black line, ○) and for coloured noise (dashed (red) black line, □) with $a_1 = 0.7$ and $a_2 = 0.702$, and for white noise (continuous (cyan) grey line, ○) and coloured noise (dashed (orange) grey line, □) with $a_1 = 0.7$ and $a_2 = 0.705$. Error bars show statistical error for some σ values. The value of R decreases due to this parameter mismatch, although the synchronization is higher for coloured noise than for white noise.

the two systems that arise because of the parameter mismatch Δa. However, for the parameter mismatch considered here and due to the common periodic forcing, the synchronized spikes (those that occur within the same period of the forcing) have approximately the same amplitude and timing. Thus, the synchronization rate R is again a good quantifier of the synchronization of the two systems in this situation.

In order to have a clearer picture of the effect of parameter mismatch, we have computed R in different situations. In figure 7, we have represented R as a function of the noise standard deviation σ for two different cases: neurons with $a_1 = 0.7$ and $a_2 = 0.702$ (black line) and neurons with $a_1 = 0.7$ and $a_2 = 0.705$ (grey line). It is observed that R is very sensitive to the presence of this mismatch parameter, but the important point is that in any case R increases as the noise increases. It is also important to note that in the presence of a parameter mismatch, coloured noise is also more effective at inducing synchronization than white noise. This result suggests that this better performance of coloured noise is robust under parameter mismatch.

In order to complete the study concerning the effect of the mismatch parameter, figures 8(a) and (b) show the synchronization rate as a function of σ and the parameter a_2, for white noise and coloured noise, respectively. We set, in all the simulations, $a_1 = 0.7$. As expected, we can observe that, for both types of noise, the synchronization rate, R, increases with σ and is drastically affected by a parameter mismatch, although for large values of noise this mismatch has less influence. This can be seen clearly also in figure 9, where a sharp drop in R value can be clearly observed when the value of $\Delta a = a_2 - a_1$ is not zero for different noise values, both for common white noise (figure 9(a)) and for common coloured noise (figure 9(b)), both with $\sigma = 0.04$ and with $\sigma = 0.1$.

Although it is possible to find systems that have a common noise, typically these systems are also affected by certain *intrinsic noise*, which we can consider white noise for simplicity and

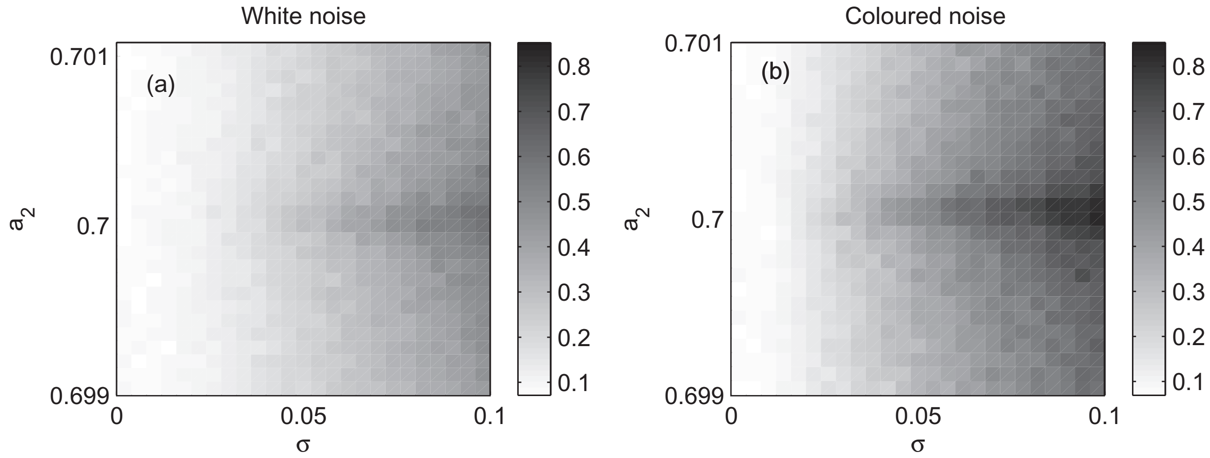

Figure 8. Synchronization rate as a function of σ and a_2 for (a) white Gaussian noise and (b) coloured noise. Darker colours indicate higher values of the synchronization rate. Notice, in both figures, the strong influence of the parameter mismatch on the synchronization rate. The optimal value of the synchronization rate takes place for $a_2 = 0.7$, as expected. We also observe, for any value of a_2, an increase in the synchronization rate insofar as the noise level increases.

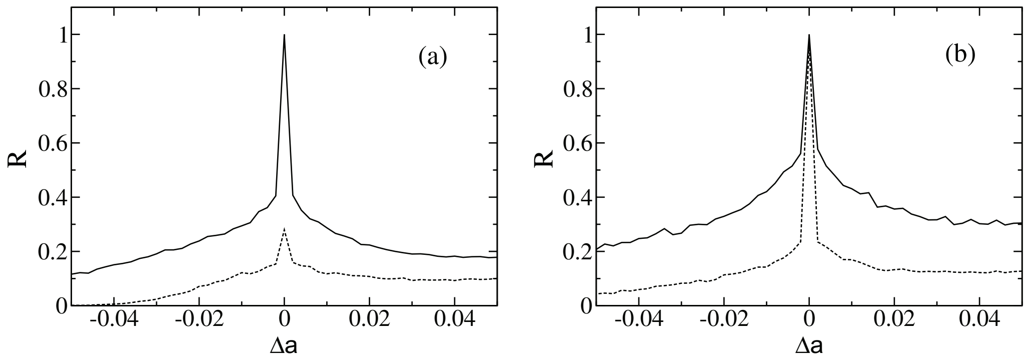

Figure 9. Variation of the synchronization rate R as a function of the parameter mismatch $\Delta a = a_2 - a_1$ for common (a) white noise and (b) coloured noise, both with $\sigma = 0.04$ (dashed line) and with $\sigma = 0.1$ (continuous line). The synchronization induced by common coloured noise is more robust under parameter mismatch.

which will typically be uncorrelated with the common noise. In order to model this situation, we rewrite the equations of the two uncoupled FHN systems in the presence of both common and intrinsic noise:

$$\begin{aligned}\frac{\mathrm{d}x_i}{\mathrm{d}t} &= c(-y_i + x_i - (x_i^3/3) + S(t) + \xi(t) + \xi_i(t)),\\ \frac{\mathrm{d}y_i}{\mathrm{d}t} &= x_i - by_i + a,\end{aligned} \tag{4}$$

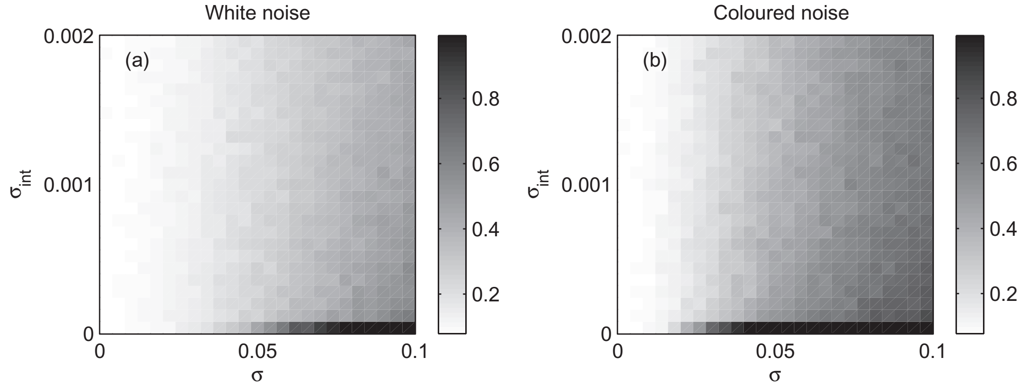

Figure 10. Synchronization rate as a function of (a) the common noise and (b) the intrinsic noise. Darker colours indicate higher values of the synchronization rate. Notice, for both white and coloured noise, the strong influence of the intrinsic noise, which causes an important decrease in the synchronization rate. We obtain the best performance for $\sigma_{\text{int}} = 0$ and large values of the common noise σ.

where $i = 1, 2$, and $\xi_i(t)$ is the intrinsic noise. This intrinsic white noise is Gaussian with $\langle \xi_i(t) \rangle = 0$ and $\langle \xi_i(t)\xi_j(s) \rangle = \delta_{ij}\sigma_{\text{int}}^2 \delta(t-s)$, where σ_{int} is the standard deviation of intrinsic noise and δ_{ij} is a Kronecker's delta. The intrinsic noise is totally uncorrelated with common noise; thus $\langle \xi_i(t)\xi(t) \rangle = 0$ for $i = 1, 2$.

The effects of the intrinsic noise are summarized in figure 10 for both white (figure 10(a)) and coloured (figure 10(b)) noise. We can see that this intrinsic noise has a negative effect on the synchronization phenomenon; hence, the bigger the value of σ_{int}, typically the smaller the value of R. Furthermore, we can notice that this intrinsic noise produces a strong decrease in the synchronization rate. Even if $\sigma_{\text{int}} \ll \sigma$, where σ is the standard deviation of the common noise (for white and coloured noise), the value of R is drastically reduced. However, we can observe that even in the presence of this intrinsic noise, by using a sufficiently strong common noise the value of R takes always values significantly bigger than those observed in the absence of common noise. These figures also show that coloured noise also induces a higher degree of synchronization than white noise, which suggests that this feature persists in the presence of intrinsic noise. The mechanism of the synchronization loss here is different from that observed in the case of parameter mismatch. The spiking rates of the two systems are now the same, but (4) can be interpreted as if we had two systems with slightly different noisy inputs, $\xi(t) + \xi_i(t)$ or $\varepsilon(t) + \xi_i(t)$. This difference is what makes the synchronized spikes more infrequent, thus decreasing the value of R.

4. Noise-induced synchronization in an electronic circuit

In order to provide further evidence of the synchronization in excitable systems induced by noise, we have implemented a laboratory version of the two FHN systems with electronic circuits. The circuit is shown in figure 11. It consists of two electronic analogue simulators coupled by means of a noise generator V_{noise} able to provide white noise and coloured noise with Gaussian distribution. The variable V_{d} is the forcing voltage amplitude, applied by means of a function generator, and V_{b} is a fixed bias voltage set to -1 V. The integrators I_1 and I_2

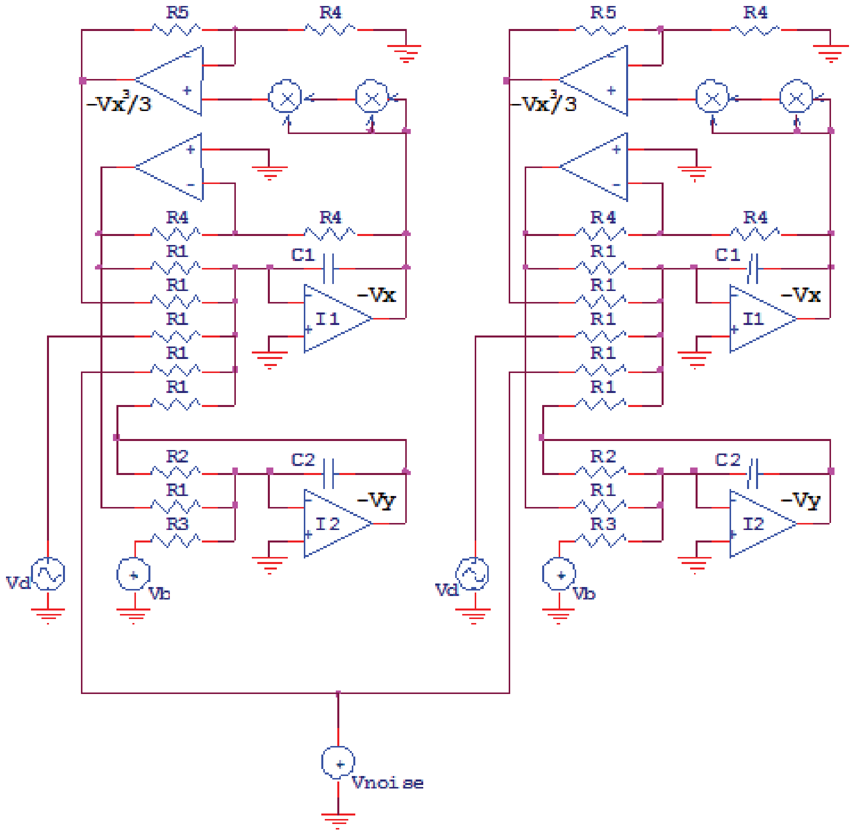

Figure 11. Layout of the electronic circuit implementing two identical FHN models, where $x \propto V_x$ and $y \propto V_y$. I: integrators; R: resistors; C: capacitors; X: multipliers; V_d: sinusoidal forcing signal; and V_b: fixed bias voltage. The numerical values are $R_1 = 100\,\mathrm{k\Omega}$, $R_2 = 125\,\mathrm{k\Omega}$, $R_3 = 143\,\mathrm{k\Omega}$, $R_4 = 1\,\mathrm{k\Omega}$, $R_5 = 1.08\,\mathrm{k\Omega}$, $C_1 = 3\,\mathrm{nF}$, $C_2 = 37.5\,\mathrm{nF}$ and $V_b = -1\,\mathrm{V}$.

have been implemented using Linear Technology LT1114CN quad operational amplifiers, while the four quadrant multipliers are Analog Devices MLT04. The acquisition of the experimental data has been performed by means of a Tektronix TDS 7104 digital oscilloscope connected to a personal computer. The voltages of the circuit V_x and V_y can be associated with x and y, respectively. The values for both, the resistors and the capacitors of this circuit, are as follows: $R_1 = 100\,\mathrm{k\Omega}$, $R_2 = 125\,\mathrm{k\Omega}$, $R_3 = 143\,\mathrm{k\Omega}$, $R_4 = 1\,\mathrm{k\Omega}$, $R_5 = 1.08\,\mathrm{k\Omega}$, $C_1 = 3\,\mathrm{nF}$ and $C_2 = 37.5\,\mathrm{nF}$, respectively.

As we have seen in previous sections, parameter mismatch has a strong influence on the phenomenon of noise-induced synchronization. Using typical electronic components, we can be sure that the system parameters have a mismatch not bigger than 0.1% of their value, yet this tiny mismatch can have a strong influence on the synchronization. On the other hand, these two electronic systems will be affected by their own intrinsic noise. Thus, in order to overcome the joint effect of parameter mismatch and of intrinsic noise and to see more clearly the effect of common noise synchronization, we opted to vary slightly the value of R_2 in one of the circuits to $R_2 \approx 126.8\,\mathrm{k\Omega}$. In this way, the spiking rates of both circuits in the absence of common noise are almost identical to each other, as it should be when the parameters are equal. Thus, we

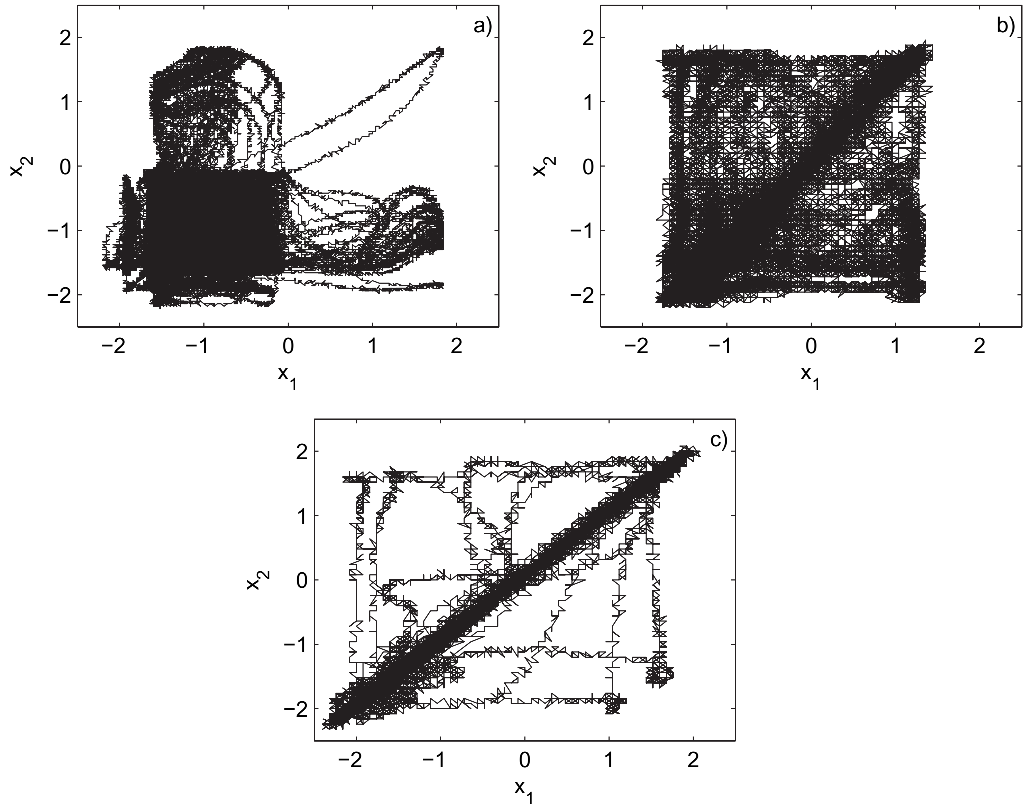

Figure 12. Experimental (x_1, x_2) diagrams for both neurons in (a) the noiseless case, (b) white noise ($R \approx 0.6$) and (c) coloured noise ($R \approx 0.9$). In the noiseless case, the neurons are not synchronized at all. The synchronization improves when noise is added to the FHN systems, but it is far from being perfect.

have decided to use this parameter mismatch in order to observe the synchronization induced by common noise in this system.

Figures 12(a)–(c) show (x_1, x_2) plots of the neurons for (a) the noiseless case, (b) white noise and (c) coloured noise, respectively. We can observe the effect of the noise on the synchronization between the neurons. This phenomenon takes place for high values of the noise amplitude and is independent of the chosen noise. We can observe better synchronization when noise is added to the FHN systems, although such a synchronization is far from being perfect. However, we can see that this synchronization is better for the coloured noise than for the white noise, in which case the synchronization between the neurons is not evident. This point is clarified by analyzing the synchronization rate R.

Figure 13 shows the synchronization rate R obtained in the experiments versus the standard deviation of noise σ, for white noise and coloured noise, respectively. The crucial importance of the noise to obtain better synchronization is noted. This figure is in good agreement with the numerical simulations shown in section 3 (see figure 6). We emphasize that the experiments carried out confirm that the synchronization rate R reached is better for the coloured noise, in which $R \simeq 0.9$ for high noise values, whereas for white noise $R \simeq 0.6$ at most. This explains why synchronization is not easily observed in figure 12(b) (white noise) and why in figure 12(c)

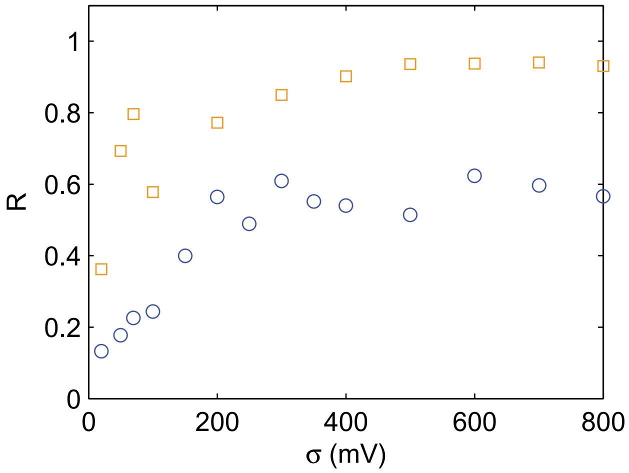

Figure 13. The synchronization rate R (see text for details) versus the standard deviation of noise σ obtained in the experiments for white noise ((blue) ○) and coloured noise ((orange) □). In both cases, the noise produces a positive effect on the synchronization between the two neurons, which is better for coloured noise.

(coloured noise) the synchronization is much better. This phenomenon is mainly due to the effect of the intrinsic noise on these systems. In any case and finally, we can summarize that, in our experimental implementation, the synchronization is very sensitive to both, the parameter mismatch and the intrinsic noise. However, as we could observe in the numerical simulations, the synchronization rate still increases as the noise level is increased.

5. Conclusions and discussion

We have studied the synchronization phenomenon in uncoupled systems due to a common noisy input, from the numerical and the experimental point of view, by taking as prototype two identical FHN systems in the presence of external noise, both white and coloured. We have implemented an electronic circuit that mimics the dynamics of these two neurons in order to check the results experimentally. We observe that the synchronization rate improves once we increase the noise level. In order to compare the effects of white and coloured noise, and at variance with previous works, we have considered the synchronization obtained for equal values of the standard deviation of the noisy perturbation that acts on the system, which allows one to make a more appropriate comparison. Our results show that at equal standard deviations of the injected noise, coloured noise gives rise to a higher synchronization degree than white noise, and we have argued that this is due to the fact that its power spectrum is higher close to the spiking frequencies of the systems. This might seem in apparent contradiction with the results provided in [23], where it was shown that white noise plays a better role in enhancing synchronization than coloured noise. However, this is due to the fact that in this paper we compare the synchronization due to the common white noise $\xi(t)$ and the common coloured noise $\varepsilon(t)$ for equal values of their standard deviation σ. In [23], however, the standard deviation σ of the white noise used to generate coloured noise with (3) was the one taken into account.

We consider that our procedure is more adequate for objective comparison, provided that we are considering the value of σ of the noisy inputs of the systems.

We have also studied the effect of a mismatch parameter between the neurons. In this case, the synchronization is very sensitive to the mismatch; however, the synchronization rate still increases as the noise level increases. Finally, we have also considered the effect of intrinsic noise different from the common noise, something that to our knowledge had not been considered in previous works on this subject, and we have observed that it plays a destructive effect on the noise-induced synchronization, although the performance of coloured noise is also better than that of white noise in these situations. Since the FHN model is a paradigmatic excitable system, we expect that our work might also be applied successfully to other systems of this kind. On the other hand, the FHN model is a good qualitative neuron model, which suggests that this work might have interesting consequences in fields where the synchronization phenomena are relevant, and can be useful for a better understanding of excitable systems that are relevant in neuronal dynamics.

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