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Effects of Intersymbol Interference on Chaos-Based Modulations

Francisco J. Escribano
Dept. Teoría Señal y Comunicaciones
Universidad de Alcalá de Henares
28805 Alcalá de Henares, Spain
Email: francisco.escribano@ieee.org

Luis López
Dept. Sistemas Telemáticos y Computación
Universidad Rey Juan Carlos
28933 Móstoles, Spain
Email: luis.lopez@urjc.es

M. A. F. Sanjuán
Departamento de Física
Universidad Rey Juan Carlos
28933 Móstoles, Spain
Email: miguel.sanjuan@urjc.es

***Abstract*—The behavior of chaos coded modulations (CCM's) in the field of digital communications has been mainly evaluated in additive white Gaussian noise (AWGN) and multipath fading channels. We study here some examples of CCM's over channels with time-invariant intersymbol interference (ISI). We provide bounds for low ISI levels and we show that these chaos-based coded systems can offer reasonable degree of robustness compared with similar coded modulation systems. We provide the analytical condition a CCM has to comply to stand a limited quantity of ISI. Our results are straightforwardly applicable to chaos-based systems of the same kind and provide a hint of the potentialities of CCM systems over dispersive channels.**

***Index Terms*—Chaos, Intersymbol interference, Modulation coding, Error analysis**

I. Introduction

In the past few years, it has been shown that it is possible to build multidimensional chaos coded modulations (CCM's) or concatenated systems based upon bad performing chaos coded modulations that, when employed in additive white Gaussian noise (AWGN) channels, can reach bit error rates (BER) comparable with other standard systems of similar complexity [1]–[3]. Recent studies point out that chaos-based modulations working at a joint waveform and coding level can be of potential interest in frequency-non selective fading channels [4]. The key to this success consists in joining the fields of digital communications and chaos theory under a common framework. This makes possible the use of well known tools from the communications field in the evaluation and design tasks of chaos-based systems [5]. Nevertheless, current literature lacks a thorough study showing what kind of channels could better match the properties of these CCM systems, since almost all the work has focused on pure AWGN or on multipath fading channels. On the other side, it is supposed that chaos-based signals in the channel should be appropriate in dispersive environments.

According to all this, in this article we address the task of showing that simple CCM systems that may perform worse than the most simple uncoded modulation in AWGN, can behave substantially better when the channel includes ISI. Thus, the chaotic signal can reveal its potential interest in broadband communications over frequency-selective time-invariant dispersive channels. We also derive bounds on the bit error probability and show that they can be tight enough to give reason of the behavior of this kind of systems. These principles can be easily extended to the whole kind of chaotic systems based on coded modulation, and help in the design and evaluation tasks, especially the condition we will establish for the existence of an error floor.

The comparison with a related standard trellis coded modulation (TCM) system [6] shows that CCM systems can keep the good properties of coded modulated signals in ISI channels. Note that our aim is not to provide methods to combat ISI impairment, but to show how CCM behave in this kind of channels and under what conditions they can be robust face to a limited level of ISI degradation. This impairment can easily appear in cable and radio systems due to the filters included at the transmitter side in order to comply with the band restrictions, or at the receiver side to reject off-band interferences [7]. Equalization is not always mandatory depending on the restrictions provided and on the margins available, so that in some systems is to be expected a certain degree of controlled ISI.

The article is structured as follows. In Section II, the communications system model and the channel model are described with the needed detail. Section III is devoted to the calculation of the BER bounds. In Section IV, we depict the simulation results together with the corresponding bounds. Section V is devoted to the conclusions.

II. System description

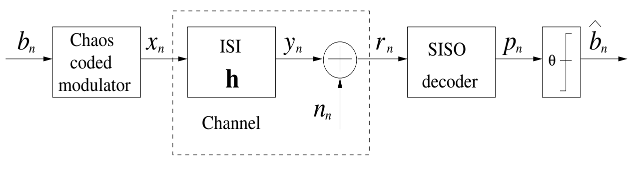

Fig. 1. Block diagram of the communications system.

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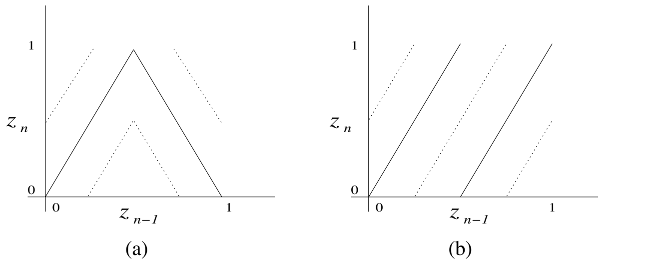

Fig. 2. Maps for the CCM systems. The continuous line corresponds to $f_0(\cdot)$; the dotted line, to $f_1(\cdot)$. (a) mTM (continuous line: TM); (b) mBSM (continuous line: BSM).

In Fig. 1 we can see the scheme of the chaos-based communications system. The chaos coded modulation (CCM) system accepts as input an identically and independently distributed (*i.i.d.*) bit sequence $b_n \in \{0,1\}$, and it produces a chaos coded modulated sequence, based on switched chaotic maps driven by small perturbations [2], following equations:

$$\begin{aligned} z_n &= f(z_{n-1}, b_n) + b_n \cdot 2^{-Q}, \\ x_n &= g(z_n) = 2z_n - 1, \end{aligned} \tag{1}$$

where $f(\cdot, 0) = f_0(\cdot)$ and $f(\cdot, 1) = f_1(\cdot)$ are chaotic maps that leave the interval $[0,1]$ invariant. They are piecewise linear maps with slope ± 2 wherever it is defined. The natural number Q is a quantization factor that indicates the number of bits used to represent z_n (and thus x_n). Note that the small perturbation manifests itself after $Q-1$ iterations. It has been shown that this kind of encoder leaves the set $S_Q = \{i \cdot 2^{-Q} | i = 0, \cdots, 2^Q - 1\}$ of 2^Q points invariant [2], so that, when taking as initial condition a value within this set (e.g. $z_0 = 0$), z_n can only take values from S_Q. In this way, we get a quantized chaotic sequence over 2^Q possible values that can be described as a trellis encoded sequence, with a state given by a shift register of Q positions and two possible transitions determined by the input bit b_n.

We shall consider the following pairs of maps $f_0(\cdot)$ and $f_1(\cdot)$:

1) Bernoulli shift map (BSM),

$$f_0(z) = f_1(z) = 2z \bmod 1. \tag{2}$$

2) Tent map (TM),

$$f_0(z) = f_1(z) = \begin{cases} 2z & 0 \le z < \frac{1}{2} \\ 2 - 2z & \frac{1}{2} \le z \le 1 \end{cases}. \tag{3}$$

3) The BSM and a shifted version of the same (multi-Bernoulli shift map, mBSM),

$$f_0(z) = 2z \bmod 1, \tag{4}$$

$$f_1(z) = \begin{cases} 2z + \frac{1}{2} & 0 \le z < \frac{1}{4} \\ 2z - \frac{1}{2} & \frac{1}{4} \le z < \frac{1}{2} \\ 2z - \frac{3}{2} & \frac{3}{4} \le z \le 1 \end{cases}. \tag{5}$$

4) The tent map and a shifted version of the same (multi-tent map, mTM),

$$f_0(z) = \begin{cases} 2z & 0 \le z < \frac{1}{2} \\ 2 - 2z & \frac{1}{2} \le z \le 1 \end{cases}, \tag{6}$$

$$f_1(z) = \begin{cases} 2z + \frac{1}{2} & 0 \le z < \frac{1}{4} \\ 2z - \frac{1}{2} & \frac{1}{4} \le z < \frac{1}{2} \\ \frac{3}{2} - 2z & \frac{1}{2} \le z < \frac{3}{4} \\ \frac{5}{2} - 2z & \frac{3}{4} \le z \le 1 \end{cases}. \tag{7}$$

In Fig. 2 we have depicted the corresponding maps. As stated, these CCM systems, when restricted to S_Q, allow an equivalent representation in terms of a *trellis encoder* [8], closely related to a trellis coded modulation (TCM) system [2]. In Fig. 3, for example, we can see the equivalent trellis encoder structure for the mTM CCM.

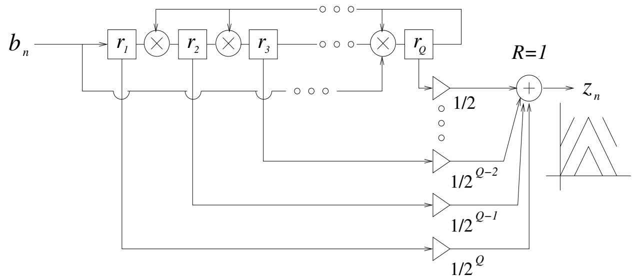

Fig. 3. Finite-state encoding structure for the mTM CCM.

The channel is an ISI channel with AWGN [7]. In digital communications, ISI impairment can easily appear due to the filters included at the transmitter side in order to comply with band restrictions, or at the receiver side to reject off-band interferences. As shown in Fig. 1, the ISI is modelated by means of a linear filter with finite impulse response $\mathbf{h} = (h_{-N}, \cdots, h_N)$, normalized to $\|\mathbf{h}\|^2 = \sum_{m=-N}^{N} |h_m|^2 = 1$, so that it does not affect the signal power at the receiver. The AWGN process adds independent Gaussian samples n_n with mean $\eta = 0$ and power σ^2. We have considered two possible impulse responses: for low ISI, and for moderate ISI. These impulse responses are shown in Fig. 4, and they have $N = 7$ coefficients. The parameter used to compare the degree of ISI is the ratio of signal power to interference power at the receiver, SIR = $h_0^2/(\|\mathbf{h}\|^2 - h_0^2)$ [7]. For low ISI we have SIR=14.84 dB, and for moderate ISI, SIR=8.88 dB.

Due to the trellis coded nature of this chaos-based signal, the receiver can be designed to decode the sequence using *maximum likelihood* (ML) or *maximum a posteriori* (MAP) sequence decoding algorithms. In this case, we have used a known MAP soft-input soft-output (SISO) decoder adapted to the decoding of this kind of chaotic sequences in AWGN channels [9], [10]. This SISO decoder, which has the advantage of allowing easy concatenation, is used here without any equalization, and thus it is simply based on the channel metrics of the AWGN channel case. Recall that we are interested in the effect of ISI mismatch in this kind of chaos coded modulations.

The SISO takes as input a block $\mathbf{r}$ of M received samples,

$$r_n = y_n + n_n = \sum_{m=-N}^{N} h_m x_{m+n} + n_n, \qquad n = 0, \cdots, M-1,$$

and produces log probability ratios $p_n = \log\left(\frac{P(b_n=1|\mathbf{r})}{P(b_n=0|\mathbf{r})}\right)$,

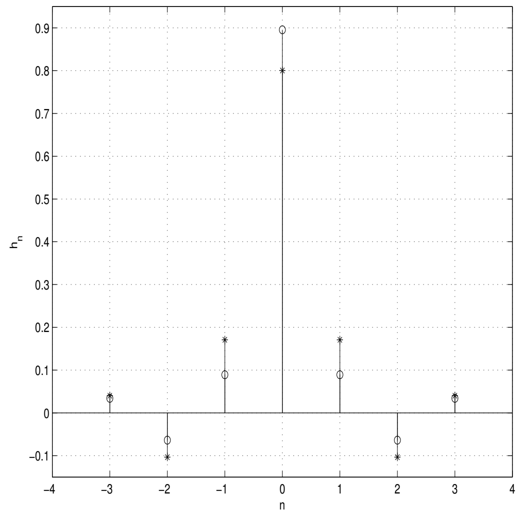

Fig. 4. Coefficients for the ISI FIR filters; 'o': low ISI; '*': moderate ISI.

which, when compared with the threshold $\theta = 0$, generate the decoded sequence $\hat{b}_n$.

III. Performance analysis

To establish comparisons with the performance of the chaos coded modulated systems, we will also take into account the case of uncoded binary phase shift keying (BPSK) over the same channel. The theoretical BER of BPSK in the ISI channel can be easily calculated if N is not large [11]. With respect to the chaos coded modulated sequence, since the *a priori* probabilities are the same, the MAP decoding is equivalent to ML decoding and the pairwise error probability can be calculated as follows. There will be an error event when, having sent the chaos coded sequence $\mathbf{x}$, the decoder chooses a sequence $\mathbf{x}' \neq \mathbf{x}$, where both sequences diverge at time m and eventually merge again after L steps. Sequences $\mathbf{x}$ and $\mathbf{x}'$ are thus related through a binary error event $\mathbf{e} = \mathbf{b} \otimes \mathbf{b}'$ containing an error loop of length L starting at time m [12]. In the case of ML decoding, and taking into account that the metrics of the SISO decoder are calculated as a function of $(r_n - x_n)^2$ [10], this is equivalent to

$$\sum_{n=m}^{m+L-1} (r_n - x'_n)^2 < \sum_{n=m}^{m+L-1} (r_n - x_n)^2. \tag{8}$$

After some algebra, we get

$$\sum_{n=m}^{m+L-1} \left[(y_n - x'_n)^2 - (y_n - x_n)^2\right] < 2 \sum_{n=m}^{m+L-1} (x_n - x'_n)^2 n_n. \tag{9}$$

For given $\mathbf{x}$ and $\mathbf{x}'$, the right hand side member of inequality (9) is a Gaussian RV, so that the pairwise error probability can be calculated straightforwardly as

$$P_e(\mathbf{x} \to \mathbf{x}'|\mathbf{x}) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{d^2_{\mathrm{ISI}}}{4P}\frac{E_b}{N_0}}\right), \tag{10}$$

where $P = 1/3$ is the power of the chaos coded modulated signal[1], and d^2_{ISI} is an *equivalent squared Euclidean distance* in the ISI channel between $\mathbf{x}$ and $\mathbf{x}'$ [13], given by

$$d^2_{\mathrm{ISI}} = \left(\frac{\sum_{n=m}^{m+L-1}(y_n - x'_n)^2 - \sum_{n=m}^{m+L-1}(y_n - x_n)^2}{d_E}\right)^2. \tag{11}$$

The factor $d^2_E = \sum_{n=m}^{m+L-1}(x_n - x'_n)^2$ is the squared Euclidean distance between sequences $\mathbf{x}$ and $\mathbf{x}'$.

According to (10), the error probability will be dominated for high E_b/N_0 by the error events leading to minimum values of d^2_{ISI}. The search for such error patterns may be a difficult task due to the structure of d^2_{ISI}, since it depends on the ISI filter coefficients and on the CCM encoding trellis. Nevertheless, in the case of low or moderate ISI, where y_n does not differ much with respect to x_n, the most probable error events are to be found among the error events with low loop lengths L. Note that, since the CCM's are nonlinear and they do not meet the uniform error event property [14], the values of d^2_{ISI} for a given binary error event $\mathbf{e}$ depend on the exact values of x_n and x'_n, and not only on $\mathbf{e}$. This makes an important difference with respect to the study of related TCM schemes under ISI impairment, where uniform error properties and linearity properties can be extensively exploited [13]

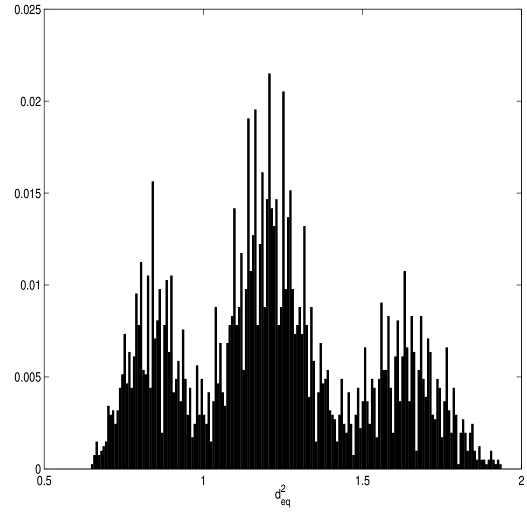

Fig. 5. Histogram of d^2_{ISI} in the case of low ISI for the BSM CCM with $Q = 5$ when the error events are of the kind described.

By performing some test simulations, it has been found out readily that the dominant error events in the low or moderate ISI cases are of the following kind:

- For the BSM CCM, the most probable error events are those with Hamming weight $w(\mathbf{e}) = 2$, length $L = Q+1$ and structure $1, 1, (Q-1)\ 0's$.
- For the TM CCM, the most probable error events are the same that give the minimum of d^2_E. They have length

[1]In fact, this is the power when $Q \to \infty$ for the CCM's proposed, since they produce data uniformly distributed in $[-1, 1]$. Nevertheless, it can be considered a good approximation for all the cases seen here.

$L = Q$, Hamming weight $w(\mathbf{e}) = Q$ and structure $1, (Q-2)\ 1's, 1$.

- For the mBSM CCM, these error events are of two kinds. The first kind is that with Hamming weight $w(\mathbf{e}) = 1$, length $L = Q$ and structure $1, (Q-1)\ 0's$. The second kind has Hamming weight $w(\mathbf{e}) = 2$, length $L = Q+2$ and structure $1, 0, 1, (Q-1)\ 0's$.
- For the mTM CCM, the error events with minimum d^2_{ISI} are the same as for the TM CCM.

As stated before, the d^2_{ISI} spectrum given by such error events depends on the ISI coefficients and on x_n and x'_n. In Fig. 5, we can see the histogram of the values of d^2_{ISI} associated to such error events in the case of the BSM CCM with low ISI. It is easy to verify that the number of possible sequences x_n, x'_n related through the binary error event $\mathbf{e}$ that can give different values for $\sum_{n=m}^{m+L-1}(y_n - x'_n)^2 - \sum_{n=m}^{m+L-1}(y_n - x_n)^2$ (and thus for d^2_{ISI}) is 2^{4N+Q+L}, so that we can evaluate exactly the associated distance spectrum for d^2_{ISI} when N, Q and L are low enough. If not, it is always possible to estimate the related histogram by taking a significant number of samples after generating sets of test values for x_n and x'_n.

In the cases considered here, the values of N, Q and L are not high and the evaluation of all the possible values of d^2_{ISI} for the mentioned binary error events is feasible. Let $D_{\mathbf{e}}$ denote the set of all such d^2_{ISI} values for a given CCM, a given ISI FIR filter and the corresponding most probable binary error event, and let D denote the number of elements in $D_{\mathbf{e}}$. Then, taking into account the expression for the error event probability (10), an average bound on the bit error probability can be calculated as [15]

$$P_b \approx \frac{w}{2D} \sum_{d^2_{\mathrm{ISI}} \in D_{\mathbf{e}}} \operatorname{erfc}\left(\sqrt{\frac{d^2_{\mathrm{ISI}}}{4P}\frac{E_b}{N_0}}\right), \qquad (12)$$

where w is the Hamming weight of the dominant binary error event $\mathbf{e}$ (i.e.: 2 in the case of the BSM CCM, and Q in the case of the TM and mTM CCM's). In the mBSM CCM case, we have two possible error events and we have to average over the two contributions, one with $w = 1$ and the other with $w = 2$.

On the other hand, we will verify that this bound does not always give reason of the bit error probability behavior, even if the binary error events actually happening are of the kind mentioned. In fact, we verify the appearance of an error floor, which can be explained looking into the expression (9). If the differences $(x_n - x'_n)^2$ are not high, and depending on the filter coefficients, there could exist the possibility that the right hand side of inequality (9) becomes negligible, specially when E_b/N_0 is higher than a threshold value (which, for fixed signal power P, means a vanishing value of the noise power σ^2). In this situation, inequality (9) becomes

$$\sum_{n=m}^{m+L-1}(y_n - x'_n)^2 < \sum_{n=m}^{m+L-1}(y_n - x_n)^2. \qquad (13)$$

For a given set of filter coefficients $\mathbf{h}$, let us denote as $B_{\mathbf{e}}$ the number of pairs of sequences $\mathbf{x}$, $\mathbf{x}'$ related through the binary error event $\mathbf{e}$ that meet inequality (13). Since there is a total of 2^{4N+Q+L} possible different values for $\sum_{n=m}^{m+L-1}(y_n - x'_n)^2 - \sum_{n=m}^{m+L-1}(y_n - x_n)^2$ when $\mathbf{x}$ and $\mathbf{x}'$ are linked by a single error event $\mathbf{e}$ of the kinds stated, and since the related $\mathbf{x}$ sequences occur with equal probability for *i.i.d.* b_n data, then the error floor associated with the binary error event $\mathbf{e}$ can be estimated as

$$P_{b_{\mathrm{floor}}} \approx \frac{wB_{\mathbf{e}}}{2^{4N+Q+L}}, \qquad (14)$$

where w takes the mentioned values depending on the CCM kind and its dominant binary error event. Again, this calculation requires a limited value for $4N+Q+L$, but there is always the possibility to bound this error floor by generating a sufficient number of test data. Note also that, for the mBSM CCM, there are two error events $\mathbf{e}$ and we should average over their corresponding contributions.

IV. Simulation Results

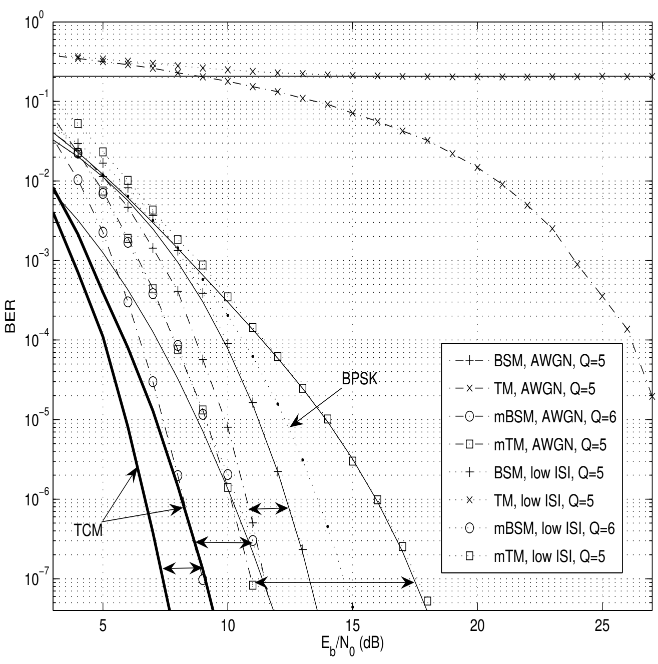

Fig. 6. Simulation results and bounds for AWGN and low ISI. Bounds are depicted with thin continuous lines. The performance of BPSK over the same ISI channel is depicted with a dotted line. The performance of the test TCM system is depicted with thick continuous lines, both for AWGN only (left) and for low ISI (right).

In Figs. 6 and 7 we can see the simulation results and the bounds for the several CCM's proposed under low and moderate ISI, respectively. We show also the theoretical bit error probability of BPSK in the same channels for comparison [11]. We have depicted also the simulation results for a conventional $R = 1$ bit/symbol TCM scheme consisting on a constraint length $\nu = 5$ encoder with polynomials 06 and 23 and quadrature phase shift keying (QPSK) modulation [15]. It is a rotationally invariant system suitable for dispersive channels. All the simulations have been run with data blocks

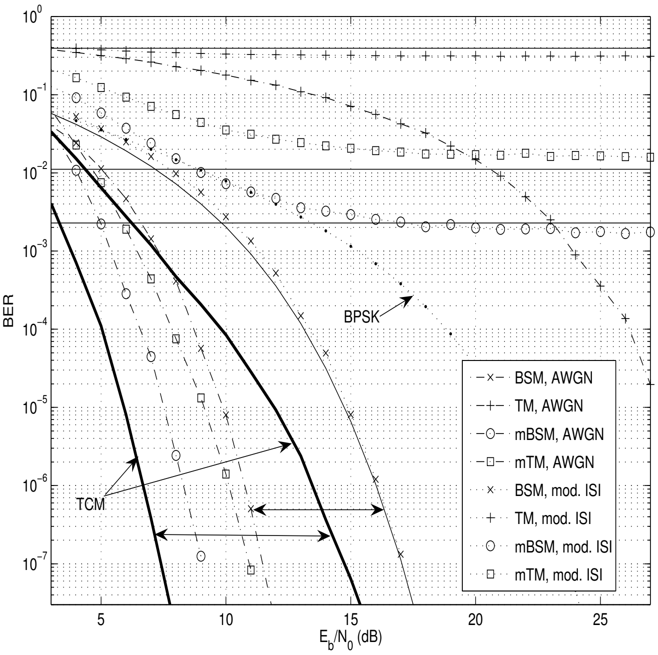

Fig. 7. Simulation results and bounds for AWGN and moderate ISI. Bounds are depicted with thin continuous lines. The performance of BPSK over the same ISI channel is depicted with a dotted line. The performance of the test TCM system is depicted with thick continuous lines, both for AWGN only (left) and for moderate ISI (right). All the cases are for $Q = 5$.

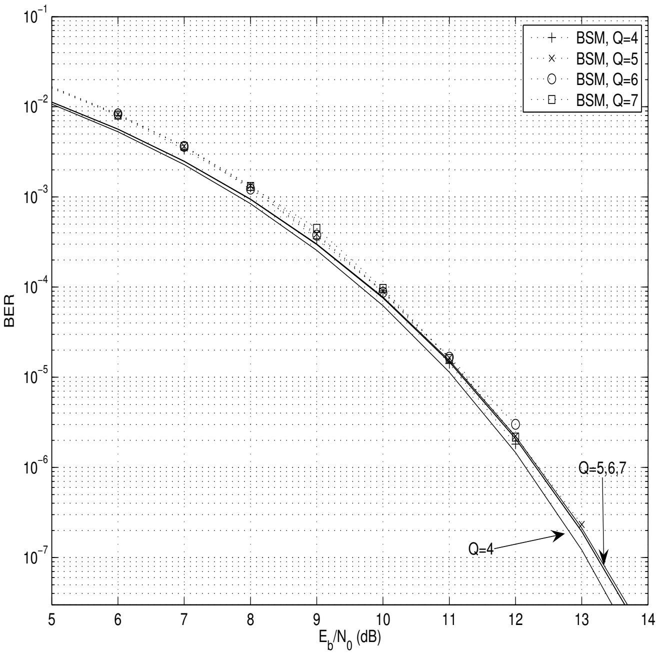

Fig. 8. Simulation results and bounds for low ISI in the case of the BSM CCM with different Q values. The bounds are depicted with continuous lines.

of $M = 10000$ bits and the BER results were recorded for 100 frames on error.

As stated for the CCM systems, in some of the cases the performance is dominated by condition (9) and the BER tends to zero as E_b/N_0 grows, while in other cases the mentioned error floor given by condition (13) is dominant for high signal to noise ratios. In low ISI, the TM CCM is severely affected by ISI impairment, while the rest of systems exhibit low or moderate losses. It is very significant that the mTM CCM comes next in E_b/N_0 loss in comparison with the AWGN channel, while the systems based on the Bernoulli shift map behave quantitatively and qualitatively better. When we have moderate ISI, however, only the BSM CCM is robust enough, keeping a steady coding gain with respect to the uncoded BPSK case, and the performance of the rest of CCM's degrades a lot, showing a high error floor and no coding gain. Note that in all the cases the corresponding bounds drawn from (12) and (14) are very tight and explain the BER behavior for high signal to noise ratios.

With respect to the conventional TCM system under low ISI, we can see that the losses in E_b/N_0 for a same BER of around 10^{-7} is similar to the losses of the mBSM CCM, and slightly higher compared to the losses of the BSM CCM, while the mTM CCM is clearly worse (see the double arrows in Fig. 6). In the case of moderate ISI (see Fig. 7, double arrows), the BSM CCM outperforms the TCM system in E_b/N_0 losses. This gives a interesting hint of how this kind of chaos-based systems could exploit both the foreseen good properties of chaos in the channel and the already known possibilities of coded modulations in dispersive environments.

The cases leading to error floors are typical cases where equalization is absolutely mandatory [16]. The results obtained stress the fact that, if condition (13) is met with a given CCM for some low loop length error event in the ISI channel, then the BER tends fast to an error floor for growing E_b/N_0. In absence of equalization, this provides a design criterion for chaos coded modulations in situations where a low degree of unequalized ISI is to be expected. Note also how the systems based on chaotic maps with poor performing abilities in AWGN (due to its poor distance spectrum, like the TM CCM [17]) also lead to bad results in ISI.

In Fig. 8, we show the results and bounds in low ISI for the BSM CCM with different quantization levels. From a value of $Q = 4$ and on, the behavior is the same, and the simulation results and the bounds are practically the same for $Q = 5, 6, 7$. This is a desirable feature in chaos coded modulations, because the encoding and decoding complexity can be kept low enough without degrading the performance, while the dynamics of the case $Q \to \infty$ can be considered for evaluation and design purposes when necessary.

V. Conclusions

Throughout this article we have studied the performance of some simple chaos coded modulations when the channel includes AWGN and low to moderate ISI impairment. We have calculated bounds for the bit error probability for the cases with limited ISI and no equalization, and they have proved to be tight enough to explain the BER behavior. We have shown that chaos coded modulations can keep the good properties of coded modulations in dispersive environments [8]. For this to happen, the distance spectrum of the original chaos coded modulation has to be robust enough to provide a good ISI distance spectrum in presence of limited ISI, as it is the case of BSM and mBSM. We have also provided a condition that could help to discard systems with potential error floors even in the presence of low ISI.

Finally, we can say that the examples proposed here provide a valuable insight into the possibilities of chaos coded modulations in frequency-selective time-invariant dispersive environments. This opens a promising way for chaos-based communications since, as the principles shown here are directly applicable to the whole kind of chaos-based systems described by a trellis, they could help to cast a theoretical ground for new and successful developments.

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