COMPLEX NETWORKS AND SOCIOECONOMIC APPLICATIONS

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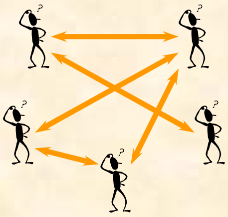
The study and characterization of complex systems is a fruitful research area nowadays. Special attention has been paid recently to complex networks, where graph and network analysis plays an important role since they reduce a given system to a simpler problem. Using a simple model for the information flow on social networks, we show that the traditional hierarchical topologies, frequently used by organizations, are poorly designed in terms of efficiency. It is also shown that this topology is stable to new relationships since it is not affected by the possibility of communication between members of the same group.

1.- INTRODUCTION

Any kind of social network may be represented by a graph in a quite natural fashion: actors are represented by vertices of the graph and the relationships among them are represented by edges.



For simplicity we assume the edges are undirected, unweighted and multiple edges between the same pair of vertices are forbidden.

Examples of social networks:
ïThe scientific collaboration network
ïThe network of sexual contacts
ïThe network of acquaintances of a person

2.- THE COORDINATION DEGREE

The *coordination degree* measures the ability of the vertices in a graph to interchange information. With the premises previously established, we define de coordination degree between two vertices *i* and *j* as:

$$\gamma_{ij} = e^{-\xi d_{ij}}$$

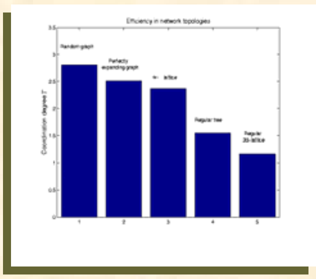
where ξ is a real positive constant called the *coordination strength* which measures the strength of the relationship.

The *average coordination degree* is defined:

Coordination degree

$$\bar{\Gamma} = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_{ij} = \frac{1}{N}\sum_{i=1}^{N}\Gamma_i$$

where *N* is the order of the graph. The average coordination degree is a global property of the social network and it can be seen as a measurement of the efficiency of the network.

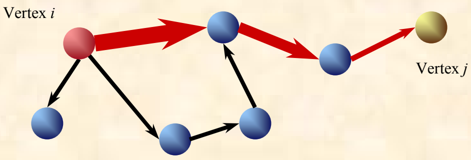


Average coordination degree for different graph structures in a social network with 400 actors.

3.- INFORMATION FLOW IN SOCIAL NETWORKS

The research in graph theory has been concentrated in modeling the spread of information from one vertex to the rest of the graph considering that the information can travel through edges without degradation in the process. However, this is not appropriate when trying to model the kind of processes that take place in collaborative social networks.

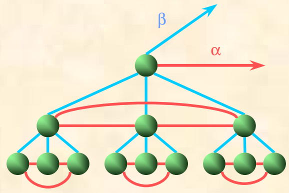
We propose a model based on two assumptions:
ïThe information travels following the shortest paths.
ïThe information coming from a vertex in the network decays exponentially with the distance to that vertex.



Path followed by the information between vertex i and vertex j. The width of the line represents the amount of information.

4.- INFORMATION IN HIERARCHICAL NETWORKS

A traditional hierarchical topology is a regular tree with degree c. This means each vertex has c-1 lower neighbors. However, we suppose that each vertex has c-1 order 1 lower neighbors and c-2 order 1 neighbors in the same level.



The edges have different coordination strength and, hence, there are two different coordination degrees. Namely, α is the coordination degree between 1 order neighbors in the same level and β between 1 order neighbors in different levels.

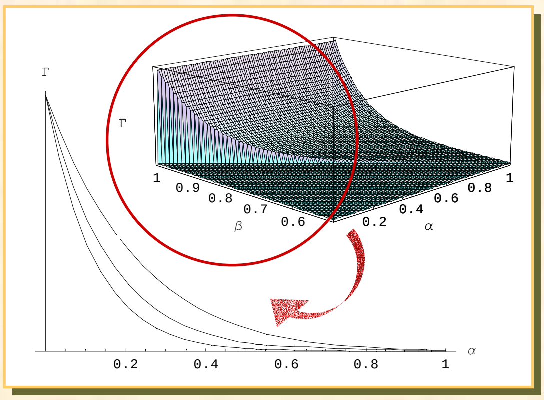
The following formula allows us to evaluate the coordination degree:

$$\Gamma_i = \begin{cases} \dfrac{(c-2)\alpha}{1-(c-1)\beta}\left\{\dfrac{\beta-\beta^{N-i}}{1-\beta} - (c-1)^{i+1}\beta^{i+2}\,\dfrac{1-\left[(c-1)\beta^2\right]^{N-i-1}}{1-(c-1)\beta^2}\right\} & \\ \quad + \dfrac{\beta-\beta^{N-i+1}}{1-\beta} + \left[1+(c-2)\alpha\right]\dfrac{1-\left[(c-1)\beta\right]^i}{1-(c-1)\beta} & i \le N-1 \\ \dfrac{1-\left[(c-1)\beta\right]^i}{1-(c-1)\beta} & i = N \end{cases}$$

The former formula is valid if $\alpha>\beta^2$. When $\alpha<\beta^2$ the shortest path is through the 1 order upper neighbor, as in a traditional hierarchical tree. The formula for $\alpha<\beta^2$ is obtained by changing $\alpha\to\beta^2$ above.

5.- RESULTS

It is reasonable to think that each actor devotes time to his neighbors proportionally to the information obtained. This gives a constraint on α and β, which implies that the maximum information is achieved when $\alpha\to 0$.



We suppose that each actor has the following constraint: (c-2)α+cβ=constant. This implies that the coordination degree is a curve on the surface defined by α and β.

6.- CONCLUSIONS

The traditional hierarchical tree is globally inefficient as compared with other topologies. However, it is a network rather spread because each actor wants to optimize his information.

Moreover, when edges between vertices with the same upper neighbor are added the information decreases. Therefore, a hierarchical tree is a stable topology against relationships between members of the same group.

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